THE SELF-CAPACITANCE OF SINGLE LAYER TOROIDAL INDUCTORS WITH FERRITE CORES

All coils have a self-resonant frequency (SRF), and as this frequency is approached the inductance and resistance increase while the Q decreases until a frequency is reached where the coil resonates in a similar way to a parallel tuned circuit. The coil thus appears to have self-capacitance and if this can be determined the changes in inductance with frequency can be readily calculated. This article derives equations for the self-capacitance of toroidal coils with ferrite cores.

1. INTRODUCTION

All coils show a self-resonant frequency (SRF) similar to a parallel tuned circuit, and because of this the accepted theory assumes that the coil has self-capacitance which along with its inductance produces this resonance. So in this theory the inductance is *constant* with frequency and the changes seen are due to the effects of a parallel self-capacitance. Further support for the self-capacitance theory is given by the fact that the measured changes are accurately modeled by a parallel tuned circuit. However, nowhere in a coil can this capacitance be measured or deduced, and this is because this capacitance does not exist and the resonance is due to standing waves on the wire, similar to a transmission-line. The inductance change is therefore a real change and not an apparent change, but nevertheless the self-capacitance model is a very useful representation. It is considered here in some detail and was found to have the form $C_{self} = A + B/N^2$ where A and B are constant for any particular size of toroid and ferrite material.

The theory here is supported by measurements, and in comparing the two it was important that the large uncertainty in ferrite permeability was eliminated, and this was done by factoring all measurements so that at the low frequencies they corresponded with the theoretical values. This is discussed in Section 9.1.

Key equations are highlighted in red, and a summary of these is given in Section 12.

This article considers coils with a toroidal ferrite core, but the author has previously considered self-resonance in cylindrical air coils (ref 1).

2. PUBLISHED MEASUREMENTS OF SELF CAPACITANCE

It is very useful in verifying a new theory to have some well documented independent experimental results. The only ones found were those carried-out by Knight (ref 2), who wound 22 coils on an Amidon FT 50-61 ferrite core (diameter of 12.7 mm) with from 4 to 36 turns.

A comparison of his measurements and the equation developed here for that core is plotted below:

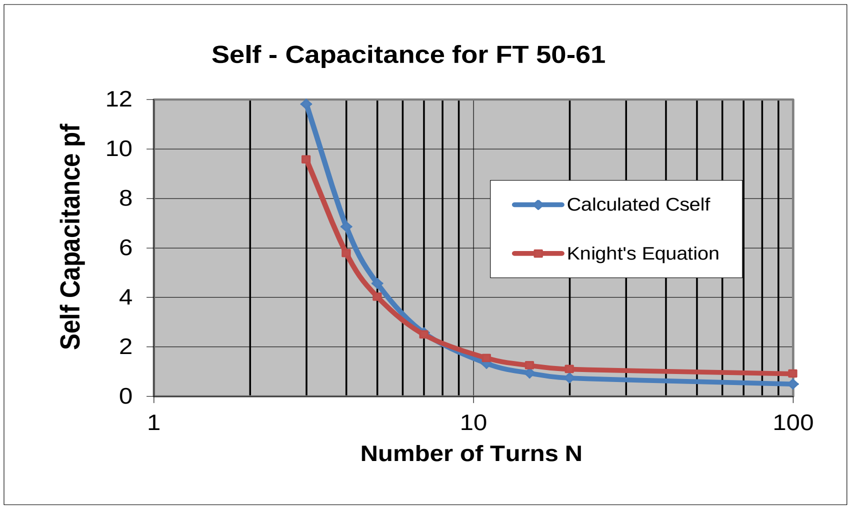

Figure 2.1 Comparison of Theory with Knight's measurements

Knight's coils all had connection leads approximately 12mm in length, and so his equation includes the capacitance of these, which is not included in the theory here. More details on the comparison are given in Section16 : Appendix 4.

3. OUTLINE OF THEORY

It is shown in this paper that self-resonance is due to a standing wave between the conductor and the surface of the ferrite, and the phase velocity of this wave is determined by the ferrite permeability and, to a lesser extent, the ferrite permittivity. From this velocity the SRF can be calculated along with the apparent self-capacitance. The change in inductance with frequency then can be determined from standard equations for lumped element circuits.

It is shown that when the permeability is constant with frequency the self-capacitance is dependent only upon the ferrite dimensions and the number of turns, and is independent of the permeability, or the diameter of the wire, or the diameter of the coil. When the permeability is not constant (the normal condition) allowance for this can be made by increasing the value of the self-capacitance.

4. SRF FROM THE TRANSMISSION-LINE MODEL

The self-resonant frequency is given by the following equation (see Appendix 3):

Self-resonant Frequency $f_r = 300 / [\, 2\, l_{path}\ (\mu')^{0.5}]$ **MHz** **4.1**

Where $l_{path} = [(N\, l_p)^2 + (\alpha\, l_e)^2]^{0.5}$
N is the number of turns
l_e **is the length of the magnetic path in metres**
l_p **is the length of the periphery of the ferrite cross section in meters**
μ' is the relative permeability of the ferrite (see below)
α (see below)

The factor α is the fraction of the toroid circumference covered by the winding. Here it is assumed that there is a 10% gap and so α=0.9 (see Section 9.2).

The equation above assumes that there is a standing wave on the conductor so that it resonates in the same way as a transmission line. Initially it was assumed that the wave would follow the conductor, but experiments show that the path length is somewhat shorter than this, and is consistent with it being the projection of the conductor onto the surface of the ferrite. For instance if the ferrite and the coil have a circular cross-section, with the coil having a larger diameter than the ferrite of course, the wave follows a helical path having the same pitch as the coil but with the diameter of the ferrite. This was well demonstrated when a coil was very loosely wound around a toroid so that its cross-sectional area was about twice that of the toroid, and the SRF was the same as for a tightly wound coil with the same number of turns. So notice that the equation above for the path length l_{path} involves only the ferrite dimensions and not the coil dimensions or the diameter of the wire (an alternative explanation is given in Appendix 3).
In the above equation the factor μ' is the ferrite permeability, commonly given graphically as below in red (this data has been extracted from the published curves) :

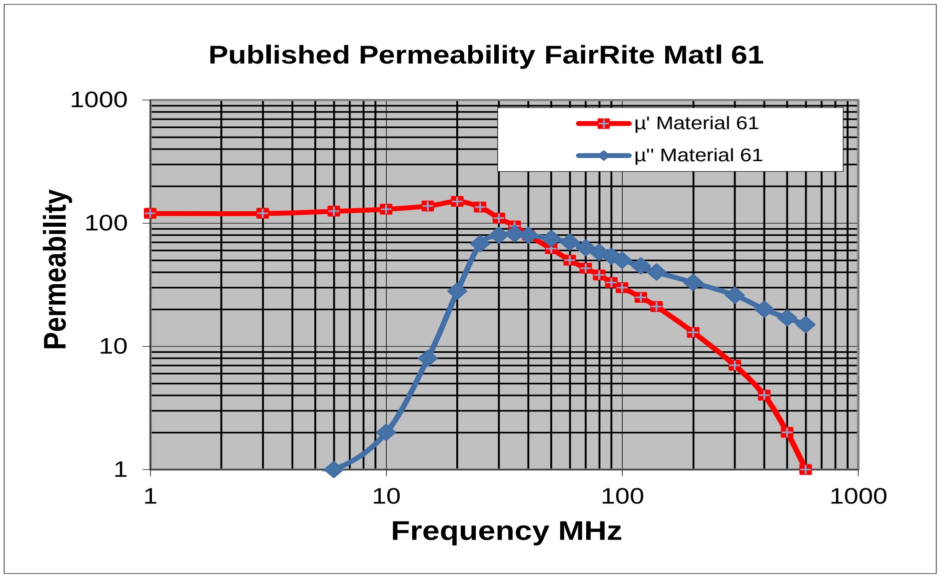

Figure 4.1 Typical Permeability Curves

Also shown is the loss component μ'', and the ratio of these is the Q of the ferrite:

$$Q_f = \mu' / \mu'' \qquad 4.2$$

The Q decreases with frequency and becomes unity at 30 MHz for this material, so the normal operating range would be up to 25 MHz. It is important to note that this is the Q of the ferrite alone, and an inductor made with this ferrite will have a lower Q due to self-resonance, and also due to the loss in the conductor, although this will often be less significant (conductor loss is considered in Section 8.2).

As with all transmission-lines a number of resonances are possible, at frequencies where the length of the line is a multiple of $\lambda_f/4$, where λ_f is the wavelength in the presence of the ferrite. Normally one end of the coil is grounded and in an air-cored coil this leads to the first resonance being $\lambda/4$, with the grounded terminal providing an effective transmission-line short circuit (ref 1). However when a ferrite core is introduced a *magnetic* short-circuit would be needed to excite the $\lambda/4$ resonance, and this is not possible. So the first resonance is $\lambda/2$. This is also the first resonance when the coil is inserted into a test jig for the measurement of inductance, resistance and the SRF.

An important consequence of the above equation is that there is not a single value for the SRF, as commonly assumed. This is because it is dependent on the ferrite parameters and these change with frequency. For instance taking a 10 turn coil wound onto a Ferroxcube TN23/14/7 core the SRF varies with frequency as follows :

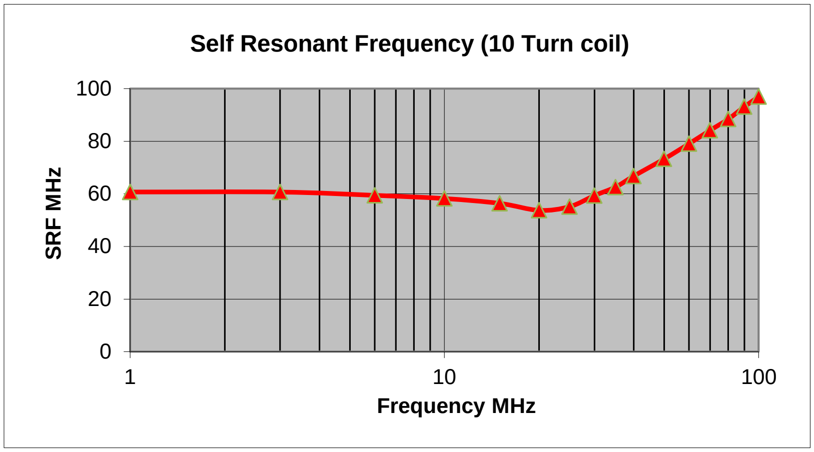

Figure 4.2 The change of SRF with frequency

This curve reflects an inverse of the permeability curve μ' such as Figure 4.1 (strictly the inverse of the square root of the permeability). So if the inductance was assumed to be constant this curve shows the self-capacitance reducing at high frequencies.

So in calculating the change in inductance at any frequency due to the SRF, the SRF f*or that frequency* must be used. For instance in calculating the inductance at 10 MHz we see from the above curve that the SRF at this frequency is 60 MHz, but when the operating frequency is raised to 60 MHz the SRF has increased to 80 MHz.

5. SELF CAPACITANCE : LOSSLESS MODEL

Self-resonance increases the apparent inductance, being higher as the self-resonant frequency is approached. This change in inductance with frequency can be modeled by a parallel circuit consisting of a perfect inductor and a perfect capacitor, as below:

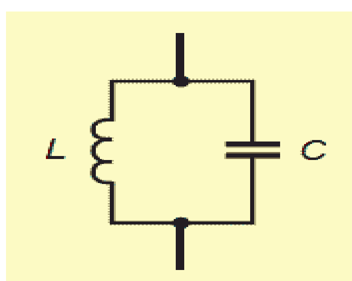

Figure 5.1 Equivalent circuit for self-resonance

In this representation the coil has a fixed inductance at all frequencies, and this inductance is equal to its low frequency value (ie low compared to its SRF). The capacitor also has a fixed value, chosen to produce a change in reactance at the terminals which matches that of the real inductor. Neither component has loss. The resonant frequency of the above circuit is :

$$f_r = 1/ [2\pi (LC)^{0.5}] \qquad 5.1$$

So the value of the capacitor C is

$$C = 1/ [(2\pi f_r)^2 L] \qquad 5.2$$

If the frequency is in MHz, the inductance in µH and the capacitance in pf this equation becomes :

Self-resonant Capacity $C_{self} = 10^6 / (2\pi f_r)^2 / L_{fo}$ pf 5.3

L_{fo} can be readily calculated from the factor A_L, given by most ferrite manufacturers, so $L_{f0}= A_L N^2$, where N is the number of turns (see also Section 9.1). The above equation then becomes :

Self-resonant Capacity $C_{self} = 10^6 / (2\pi f_r)^2 / (A_L N^2)$ pf 5.4

where f_r is the resonant frequency in MHz given by Equation 4.1
A_L is in $\mu H/N^2$

6. SELF CAPACITANCE WHEN PERMEABILITY IS CONSTANT

This section derives an equation for the self-capacitance when the permeability is constant with frequency. Inserting the equation for the resonant frequency, Equation 4.1, into that for the self-capacitance, Equation 5.4, gives :

$$C_{self} = 10^6 \mu_f [(N l_p)^2 + (\alpha l_e)^2] /[(2\pi)^2 300/2)^2 A_L N^2]$$

$$= 1.13 \mu_f [(N l_p)^2 + (\alpha l_e)^2]/ [A_L N^2]$$

For lengths in mm and A_L in $\mu H/N^2$, this becomes :

$$C_{self} = 1.13\ 10^{-6} \mu_f l_p^2 / A_L + 1.13\ 10^{-6} \mu_f [(\alpha l_e)^2/ A_L] / N^2 \qquad 6.1$$

where l_p is the length of the periphery of the cross-section in mm
μ_f is the low frequency relative permeability of the ferrite
α is the proportion of ferrite with winding (ie 0.9)
l_e is the length of the magnetic path in mm
A_L is in $\mu H/N^2$
N is the number of turns

This equation has been written in the form $C_{self} = A + B/N^2$ and the reason for this will become clear in Section 7 where it is shown that the effects of changing permeability can be allowed-for with the addition of another term in $1/N^2$.

As an example, the Amidon FT 50-61 toroid has A_L =69/1000, l_e =30.6 mm, l_p = 15.35 mm, and μ_f=120, giving the self-capacitance as :

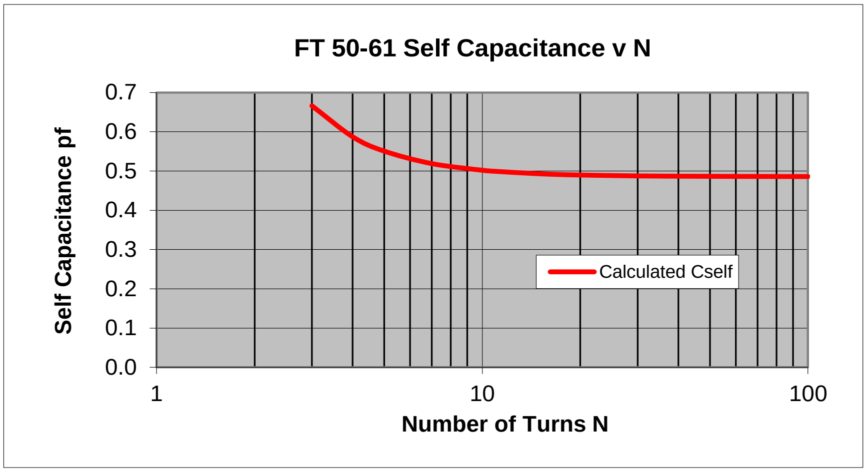

Figure 6.1 Calculated Self Capacitance of FT50-61 core

The overall reactance of the circuit Figure 5.1 can now be calculated from normal circuit theory using the value of C_{self} given by the above equation:

$$X_t = -X_L\, X_C/(X_L - X_C) \qquad 6.2$$

At frequencies below the resonant frequency this circuit is inductive, with an apparent inductance L_{app} given by :

$$L_{app} = X_t / \omega = -\, X_L\, X_C/(X_L - X_C)/\, \omega \qquad 6.3$$

where $X_L = \omega L_{fo} = \omega\, A_L\, N^2$
$Xc = 1/(\omega C_{self})$
C_{self} given by Equation 6.1

For the FT50-61 toroid with 11 turns L_{fo} =8.97 µH and the self-capacitance is 0.7 pf (Equation 6.1). The apparent inductance (Equation 6.3) is then, in red:

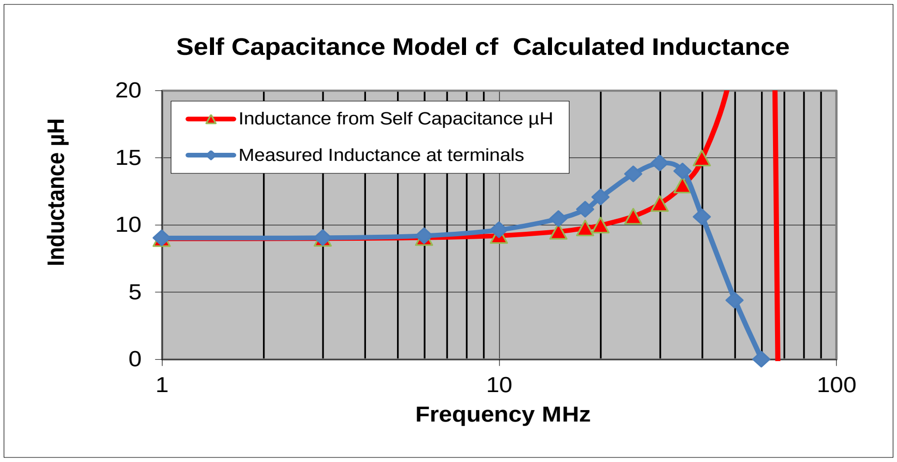

Figure 6.2 Self Capacitance Model cf Measured Inductance

Also shown in blue are the measured values. Agreement is very poor and this is because the above analysis assumes that the permeability is constant at its low frequency value, whereas it changes considerably.

A method for allowing for the changing permeability is given in the next section.

7. CORRECTING FOR PERMEABILITY CHANGES

7.1. Introduction

The equations given so far correctly predict the SRF (see Figure 6.2) but do not adequately describe the change in inductance with frequency, and this is because the permeability is not constant. The change in permeability can be quite large but is somewhat disguised by the usual logarithmic representation (Figure 4.1), and is shown more clearly if plotted linearly as below:

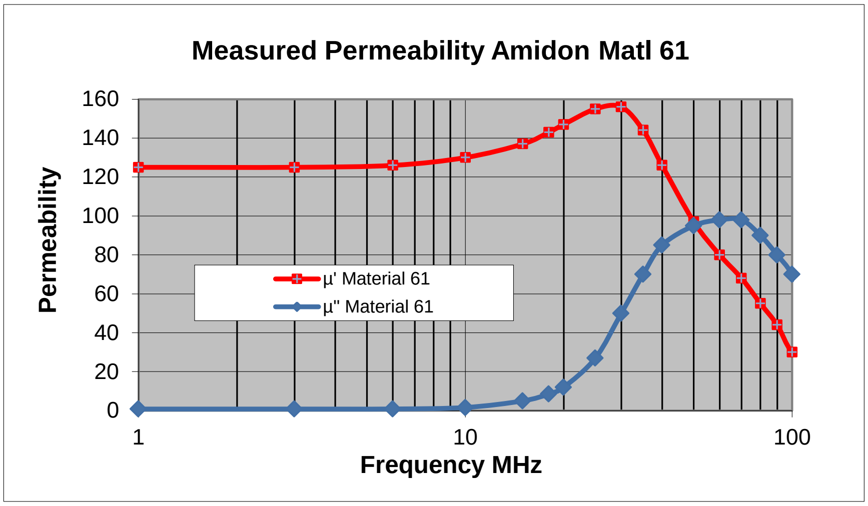

Figure 7.1.1 Self Capacitance Model cf Measured Inductance

The above detail is difficult to obtain with any accuracy from the manufacturer's logarithmic curves, and so it is from the author's own measurements on the Amidon FT 50-61 core (see Appendix 1).
Given that the apparent inductance L_{app} is defined by Equations 6.1 and 6.3, modification of either of these equations is an option for allowing for changes in permeability. So two methods are given : the first method increases C_{self} (Equation 6.1) in order to reduce the resonant frequency, and this is described below. The second method modifies Equation 6.3 with a frequency dependent value of inductance and this is described in Appendix 6.

7.2. Increased C_{self} to allow for changing permeability

The first method is to increase the value of C_{self}, so that the SRF is reduced. For instance taking the FT50-61 core with 11 turns, if the capacitance is increased from 0.7 pf to 1.34 pf the SRF reduces from 80 MHz to 45 MHz, and the correlation with measurements improves considerably over the useful range to 25 MHz (compare with Figure 6.2) :

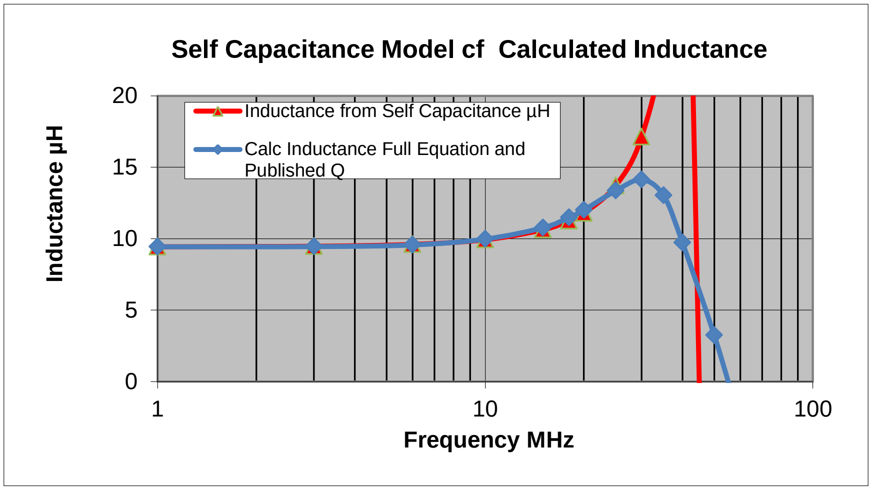

Figure 7.2.1 Self Capacitance Model and Measured Inductance Matl 61

Clearly for this optimum match the new resonant frequency f_r' must now be related to the frequency at which the permeability peaks f_p (figure 7.1.1), and in general we can say $f_r' = k\ f_p$. An additional capacitance must be added to the self-capacitance and this must be chosen to resonate at frequency f_r' with the inductance L_{fo}, and this gives a value of $C = 10^6/[(2\pi\ k\ f_p)^2\ L_{fo},]$, for f_p in MHz and L_{fo} in µH.
Including the self-capacitance C_{self} (Equation 6.1) and noting that $L_{fo} = A_L N^2$, this gives :

$$C'_{self} = 1.13\ 10^{-6}\ \mu_f [\ l_p^{\ 2} / A_L] \ + \ 1.13\ 10^{-6}\ \mu_f [\ (\alpha\ l_e)^2 / A_L] / N^2 \ + \ 10^6/[(2\pi\ k\ f_p)^2\ A_L\ N^2] \quad pf \qquad 7.2.2$$

where l_p is the length of the periphery of the cross section in mm
μ_f is the low frequency relative permeability of the ferrite
α is the proportion of ferrite with winding (e.g. 0.9)
l_e is the length of the magnetic path in mm
A_L is in µH/N^2
N is the number of turns
f_p is in MHz
k =2 for material 61 (see below)

Notice that the first term is dependent only on the ferrite parameters and independent of N, whereas the second and third terms reduce as N^2.
For the Amidon FT50-61 toroid (Material 61) μ_f = 125, k=2 (see below), f_p =30 MHz, A_L= 0.069 µH/N^2, l_p = 15.35 mm, l_e =30.6 mm and then Equation 7.2.2 gives:

$$C'_{res} = 0.49 + 1.8/N^2 + 104/N^2 \qquad pf \qquad 7.2.3$$

This equation is plotted below.

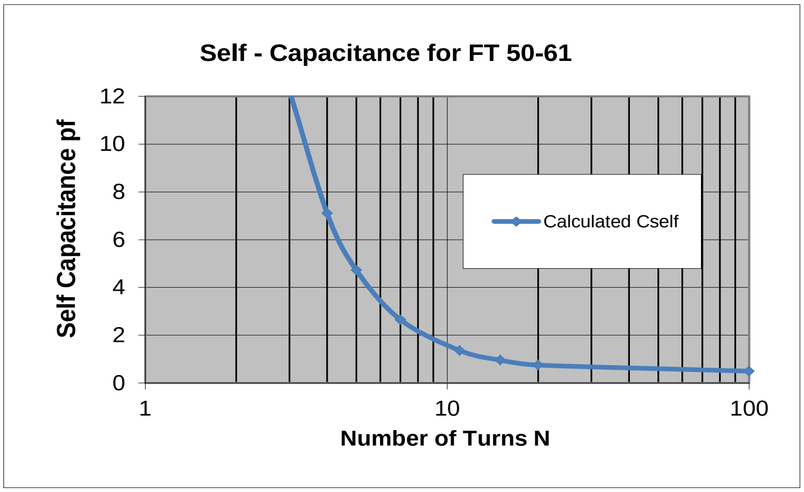

Figure 7.2.2 Self Capacitance Model for FT 50-61

The factor k was found by matching graphically the calculated change in inductance with frequency, using the value of C_{self} given by 7.2.2, with the measured inductance. For material 61, a Ni-Zn material, a value of k=2 was optimum, however the same factor also gave a good match with Mn-Zn material 77, as shown below. Given that these are very different ferrites it suggests that a factor of k=2 may be applicable to all ferrites.

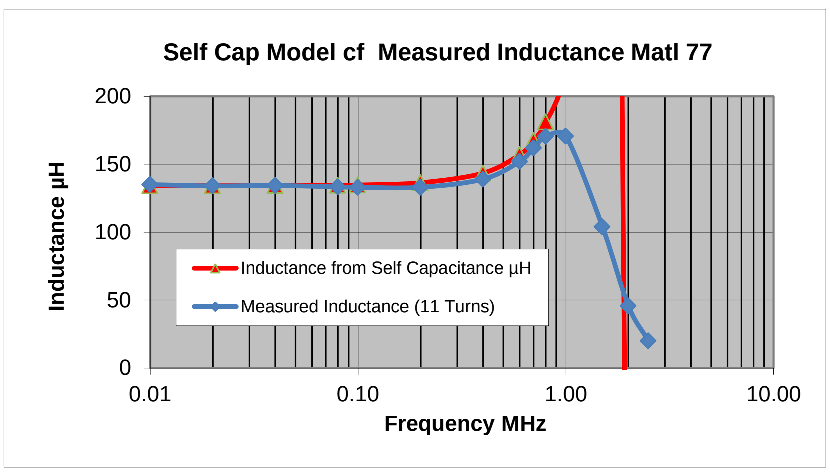

Figure 7.2.3 Self Capacitance Model and Measured Inductance Material 77

7.3. Simplification of Equation 7.2.2

The middle term of Equation 7.2.2 generally contributes less than 6% to the value of C'_{self} and therefore can often be ignored (see Appendix 8), as can be seen by Equation 7.2.3. The exception is when the permeability is below 16 or the number of turns is less than 5 when the middle term should be used. In addition, the first term above can be expressed in terms of the toroid dimensions only (Appendix 5) to give :

$$C'_{self} \approx [1.13\ 10^{-3}\ l_e\ l_p^{\ 2}\ ] / [0.4\ \pi\ A_e)] \ +\ 10^6/[(2\pi\ k\ f_p)^2\ A_L\ N^2]pf \qquad 7.3.1$$

where l_p is the length of the periphery of the cross section in mm
l_e is the length of the magnetic path length in mm
μ_f is the low frequency relative permeability of the ferrite
A_L is in $\mu H/N^2$
N is the number of turns
f_p is the frequency in MHz at which μ_f peaks
Equation is valid for $N \geq 5$ AND $\mu \geq 16$

If the above equation is expressed as $C'_{self} = A + B/N^2$, the uncertainty due to the manufacturer's tolerance on the ferrite parameters is $C'_{self} = A \pm 12\% + (B/N^2) \pm 20\%$ (see Appendix 7).

It is emphasised that Equations 7.2.2 and 7.3.1 are used in conjunction with Equation 6.3 unmodified, to give the overall apparent inductance :

$$L_{app} = X_t / \omega = - X_L X_C/(X_L-X_C)/ \omega \qquad 7.3.2$$

where $X_L = \omega L_{fo}$
$X_c = 1/(\omega C'_{self})$
C'_{self} given by Equation 7.2.2 or 7.3.1

7.4. Examples

The following are examples of the self-capacitance as calculated from Equation 7.3.1. A core size of OD 12.7 mm, ID 7.15 mm, and height 4.78 mm has been used along with Fair-Rite data on the permeability, and the A_L value for this core size and permeability. The frequency at which the permeability peaks, f_p, has been read from the published curves of permeability.

Material		μ_f	$A_L(\mu H/N^2)$	f_p (MHz)	C'_{self} (pf)
68	Ni-Zn	16	0.00864	210	$0.47 + 17/N^2$
67	"	40	0.0022	80	$0.47 + 45/N^2$
61	"	125	0.069	20	$0.47 + 229/N^2$
52	"	250	0.148	8	$0.47 + 702/N^2$
77	Mn-Zn	2000	1.180	0.8	$0.47 + 8385/N^2$

Notice that the first term in C'_{self} is constant at 0.47 pf, independent of material, and this because it is determined only by the toroid dimensions (see Appendix 5).

7.5. Possible Universal equation for C'_{self}

An equation is developed here for the self-capacitance which might be universal for all modern ferrites, and requires only the dimensions of the toroid to be known along with its low frequency permeability.
In Equation 7.3.1 the factors f_p and A_L are not independent, because there is a close relationship between f_p and μ, and A_L is directly proportional to μ. Snoek in 1948 showed that the product of f_p and μ are nearly constant (see Hamilton ref 3). This relationship is most strikingly demonstrated by the following curves from Fair-Rite, for four of their materials (ref 4).

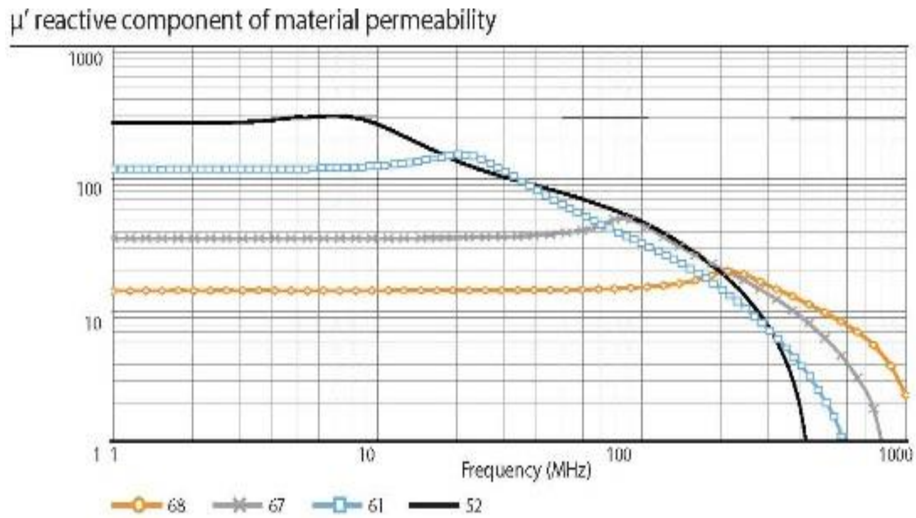

Figure 7.5.1 Relationship between f_p and μ

These curves show the relationship between f_p and μ, to be :

$$f_p = K / [\mu_f^{\,x}] \quad \text{MHz} \qquad 7.5.1$$

where K= 5300 and x = 1.16 (for Fair-rite materials)

Substituting this equation into the second term of Equation 7.3.1, and noting that $A_L = \mu_o \mu_f A_e / l_e = 0.4\,\pi\, 10^3\, \mu_f A_e / l_e$ where A_L is in μH/N², and A_e is in mm² and l_e in mm.

$$10^6/[(2\pi\, k\, f_p)^2 A_L N^2] = 10^9 \mu_f^{(2x-1)} l_e / [(2\pi\, k)^2 K^2\, 0.4\pi\, A_e N^2]$$
$$= 10^9 \mu_f^{(2x-1)} l_e / [\, 49.6\, k^2 K^2 A_e N^2]$$

Equation 7.3.1 then becomes :

$$\mathbf{C'_{self} \approx [1.13\; 10^{-3}\, l_e\, l_p^{\,2}\,] / [0.4\,\pi\, A_e)] \;+\; 10^9\, \mu_f^{(2x-1)}\, l_e \,/\, [\,49.6\, k^2 K^2 A_e N^2] \quad pf \qquad 7.5.2}$$

where l_p is the length of the periphery of the cross section in mm
l_e is the length of the magnetic path length in mm
μ_f is the low frequency relative permeability of the ferrite
N is the number of turns
A_e is the toroid cross-sectional area in mm²
For Fair-Rite materials x=1.16, k=2 and K =5300

Notice that the only ferrite parameters which are needed are its dimensions and its low frequency permeability. The factors x, k and K have been determined for the Fair-Rite materials but it is possible that they are common to all modern ferrites. If so, the above equation may be generic and applicable to ferrites from any manufacturer.

If the above equation is expressed as $C'_{self} = A + B/N^2$, the uncertainty for Fair-Rite materials due to the tolerance of the ferrite parameters is $C'_{self} = A \pm 11\% + (B/N^2) \pm 35\%$ (see Appendix 7)

7.6. The Effect of Ferrite Tolerances

We now have two equations for calculating the self-capacitance, Equations 7.3.1 and 7.5.2. For the toroid given in Section 7.4 these give the following values :

Material		μ_f	C'_{self} Equation 7.3.1	C'_{self} Equation 7.5.2
68	Ni-Zn	16	$0.48 + 17/N^2$	$0.47 + 15/ N^2$
67	"	40	$0.47 + 45/N^2$	$0.47 + 50/N^2$
61	"	125	$0.47 + 229/N^2$	$0.47 + 218/N^2$
52	"	250	$0.46 + 702/N^2$	$0.47 + 538/N^2$
77	Mn-Zn	2000	$0.44 + 8385/N^2$	$0.47 + 9341/N^2$

The differences are due to the tolerances in the manufacturer's data.
So what effect would these tolerances have? Taking the Example in Figure 7.2.1, if the capacitance is increased by 20% the correlation with measurements changes to :

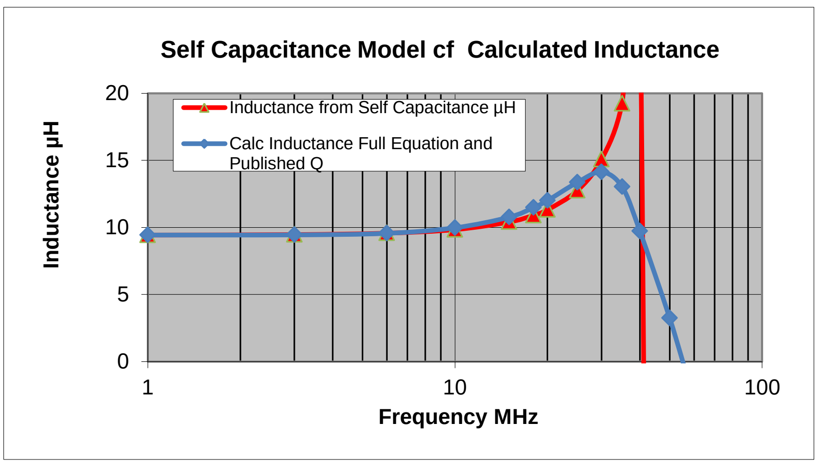

Figure 7.6.1 The Effect of 20% increase in C'_{self} (compare with Figure 7.2.1)

The correlation degrades by only 2% at frequencies up to 20 MHz, and so the calculation is relatively insensitive to the manufacturing tolerances.

8. LOSSES IN THE SELF CAPACITANCE MODEL

8.1. Losses in the Ferrite

Ferrite loss increases with frequency, as shown below by the reducing Q_f of the Fair-Rite Material 61 (where $Q_f = \mu'/\mu''$ has been derived from the published curves reproduced in Figure 4.1):

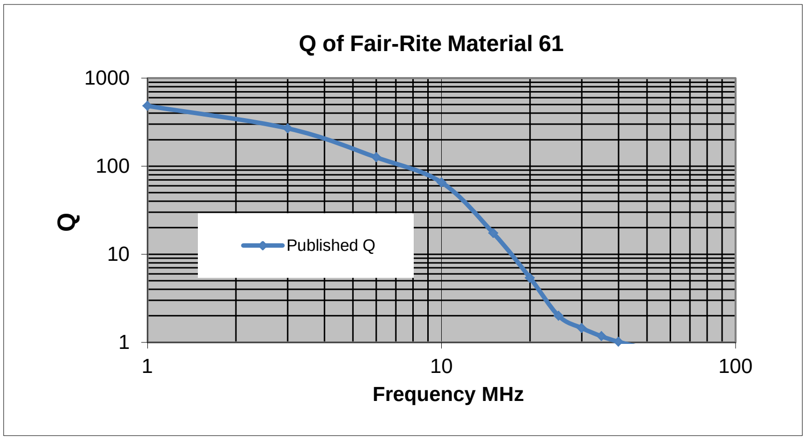

Figure 8.1.1 The Q of Fair-Rite Material 61

NB the value of μ'' below 6 MHz is not given by Fair-Rite and so the data at the lower frequencies is from measurements taken by the author

To account for this loss the model Figure 5.1 could include a parallel resistance R_p :

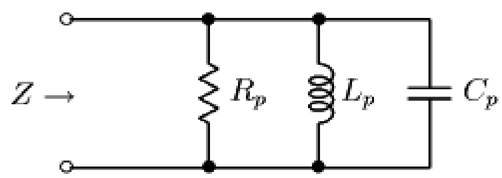

Figure 8.1.2 Self-capacitance model with Loss

The Q is then be given by (ignoring the effects of self-resonance for now):

$$Q_R = R_p /(2 \pi f L) \qquad 8.1.1$$

R_p must be chosen to make this Q equal to that of the ferrite (μ'/μ''), so :

$$R_p = (2 \pi f L) (\mu'/\mu'') = 2 \pi f N^2 A_L (\mu'/\mu'') \qquad 8.1.2$$

where f is in MHz
A_L in μH/N^2

In the low frequency region the ferrite Q decreases as roughly 1/f, so that (μ'/μ'') = ψ/f, where ψ is to be determined for each ferrite from its permeability curves. So :

$$\mathbf{R_p = 2 \pi N^2 A_L \psi} \qquad \mathbf{8.1.3}$$

For the Fair-Rite material 61, we find from Figure 8.1.1 that ψ = f (μ'/μ'') = 660 (ie at 10 MHz, μ'/μ'' = 66), and the Q calculated from Equations 8.1.1 and 8.1.3 is then :

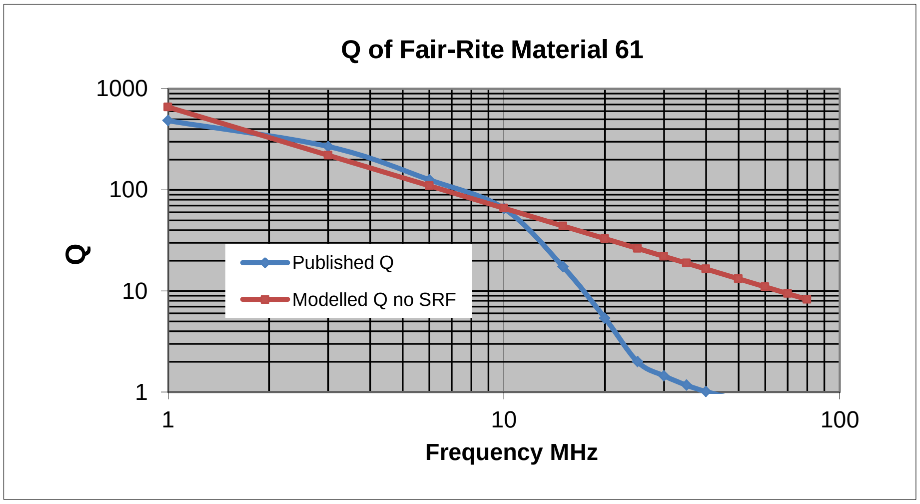

Figure 8.1.3 The Q with parallel resistance R_p

The model is good up to about 10 MHz, but above that frequency the ferrite Q decreases dramatically, and this simple model with a fixed value of R_p is no longer valid.
The analysis so far has not allowed for the effects of self-resonance, and if this is included Equation 8.1.1 becomes (see Section 10):

$$Q_R = R_p [1- (f/f_r)^2]/[2 \pi f N^2 A_L] \qquad 8.1.4$$

f_r is the self-resonant frequency in MHz given by Equation 4.1
A_L is in μH/N^2

When the number of turns is small, f_r is large and the factor $[1- (f/f_r)^2]$ is close to unity, and the effect of self-resonance is small. However when the turns are large it is more significant, and for instance with 55 turns wound onto a Fair-Rite Toroid which has dimensions of OD 12.7 mm, ID 7.15 mm, and height 4.78, material 61, f_r = 16 MHz, and Equation 8.1.4 gives:

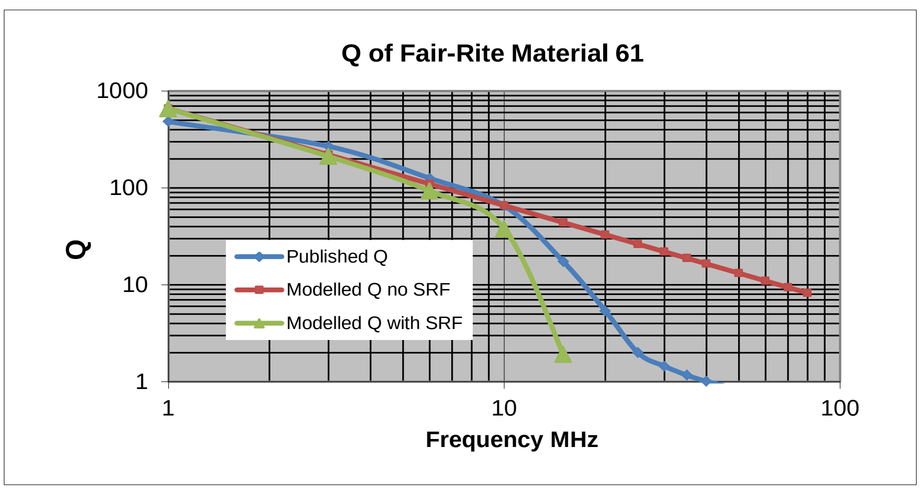

Figure 8.1.4 Modelled Q with effects of SRF (Material 61)

It should be appreciated that the calculated reduction in Q above 6 MHz (green curve) assumes that the ferrite Q decreases as per the red curve (ie R_p is constant), and so the real decrease above 10 MHz is somewhat greater than that shown by the green curve.

For the Fair Rite material 77 (Mn-Zn), $\psi = f(\mu'/\mu'') = 3$ (ie at 0.1 MHz, $(\mu'/\mu'') = 30$), and for the same size core the Q calculated from Equations 8.1.3 & 8.1.4, with 20 turns ($f_r = 1.63$ MHz) is:

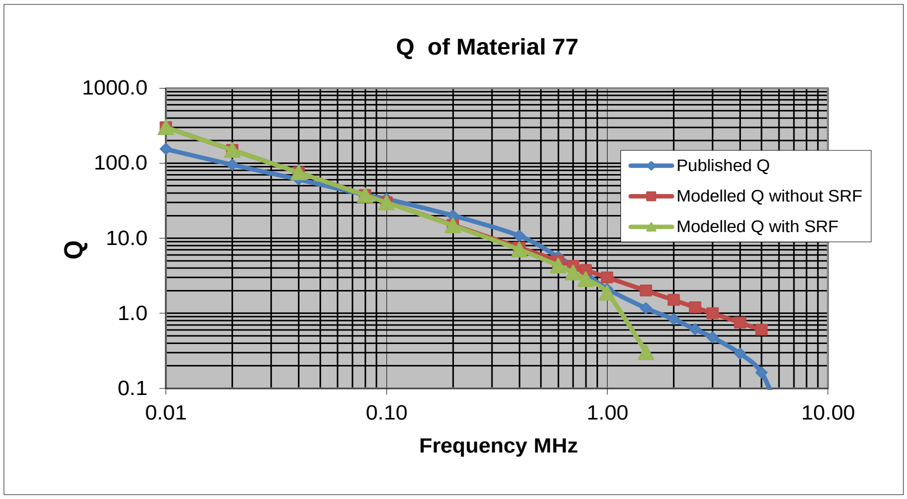

Figure 8.1.5 Modelled Q with effect of SRF (Material 77)

8.2. Conductor Losses

The resistance of a conductor is given by $R = \rho\, l / A$, where ρ is the resistivity of the conductor, l is its length, and A its cross-sectional area. At high frequencies the current flows only in a thin skin on the surface of the conductor and the equation becomes, for a straight conductor, with a circular cross-section:

$$R_o = \rho\ l / (\pi\, d_w\, \delta) \qquad 8.2.1$$

where d_w is the wire diameter
δ is the skin depth = $66.6 / \sqrt{f}$ mm for copper
f is in hertz.

When the wire is wound into a coil the resistance increases due to the proximity effect of adjacent turns. This effect can be estimated from the increase in resistance of a number of parallel wires, given by (reference 5) :

$$R/R_o \approx 2.2 / [1 + 1.35/n] \qquad 8.2.2$$

where n is the number of wires
.

The above equation is for wires close and touching, and a lower ratio will be given when the wires are spaced, approximately by the ratio $[1 + 1/ (2\, (p/ d_w)^2\ + 1)]/1.33$, (also reference 5) where p is the distance between wire centres, and d_w the wire diameter.
For coils with air cores there is a further increase in resistance due to flux from the centre of the solenoid 'leaking' into the coil especially at the ends (see Payne ref 6). With a ferrite core having a high permeability there is very little leakage and it is assumed that this effect can be ignored.
For the coils considered here the loss due to the conductor is small and generally the ferrite loss dominates. The conductor resistance increases as $\sqrt{f}$ (Equation 8.2.1) whereas the ferrite loss increases as f, and so the copper losses will be a higher fraction of the total loss at the lower frequencies. For instance 10 turns of copper wire with a diameter of 1 mm wound onto a a Fair-Rite Toroid 5961001801 (OD 22.1mm) will have a resistance of 0.04 Ω at 1 MHz, assuming the proximity effect is 1.6. The reactance at 1 MHz is 52 Ω

so if the conductor loss were the only loss the Q would be 52/0.04 = 1300. The Q of the ferrite at this frequency is 480 and so the combined Q is $Q_1Q_2/(Q_1+Q_2)$ = 351. The conductor resistance thus gives a 27% decrease in the overall Q. But note that this decrease is from a high value of 480, and a larger wire diameter would reduce the conductor loss below that calculated.
At the higher frequency of 10 MHz, the conductor resistance is 0.16 Ω assuming the proximity effect is 1.6. The reactance is 520 Ω so if the conductor loss were the only loss the Q would be 520/0.13 = 4000. The Q of the ferrite at this frequency is 82 and so the combined Q is $Q_1Q_2/(Q_1+Q_2)$ = 80.4. The conductor resistance thus gives only a 2% decrease in the overall Q at this higher frequency.
The above analysis is not thorough but is designed to give a guide on the effect of conductor loss.

9. OTHER ISSUES RELATED TO SELF CAPACITANCE

9.1. Low Frequency Inductance and A_L

The low frequency inductance of a ferrite toroid is given by :

$$L = \mu_o\mu_r N^2 A_e / l_e \qquad 9.1.1$$

where A_e is the cross-sectional area of the ferrite
l_e is the mean length of the magnetic path

The factor $\mu_o\mu_r A_e / l_e$ is given by the ferrite supplier as A_L, and so $L = N^2A_L$. However the inductance calculated this way can differ significantly from the measured value, and this is because the tolerance on A_L is large, and typically ± 20%. In comparing theory with experiments it was important to eliminate this error and so in all cases the calculated inductance was factored to make the low frequency inductance agree with measurements. Thus it was assumed that the low frequency measurement is accurate and free from any effects of self-resonance. If A_L is not quoted it can be calculated from :

$$A_L = \mu_0\, \mu_f\, A_e / l_e \quad \text{H/N}^2 \qquad 9.1.2$$

where $\mu_0 = 4\pi\ 10^{-7}$,
μ_f is the relative permeability of the ferrite
A_e is the cross-sectional area of the toroid, in m^2
$l_e = \pi D_f$, equal to the mean circumference of the toroid in meters

If A_L is in μH/N^2 , A_e is in mm^2 and l_e is in mm, this becomes:

$$A_L = 0.4\,\pi\, 10^{-3}\, \mu_f\, A_e / l_e \quad \mu\text{H/N}^2 \qquad 9.1.3$$

9.2. Gap Capacitance

Most authors say that the ends of the winding should be not too close together so as to minimise the capacitance across the gap, and a gap of around 10% of the toroid periphery is often recommended.
In a test the author reduced the gap from around 13% to around 5%, and the measured SRF reduced by only 3.5%. The 5% gap in fact represented a full winding since there were 20 turns and so this ‘gap’ equalled the pitch of the winding.
For the toroids measured here the gap was about 10% for all windings, and good agreement with the theory was obtained with no allowance for gap capacitance. However, this good agreement did depend upon having a low capacitance test jig with its stray capacitance carefully calibrated-out (see Section 11.2).

9.3. Capacitance between wire leads

The toroids measured here had short connecting leads of length around 3mm for each of the two leads. In practice longer leads may be necessary and these will introduce additional capacitance and reduce the SRF. This lead capacitance will have to be added to the self-capacitance calculated by the equations here.

Initially an attempt was made to measure the capacitance between two 12 mm long leads of diameter 1mm with a spacing of around 4mm and this gave a capacitance averaging 0.25 pF over the frequency range 100 to 200 MHz. However this small value is very difficult to measure accurately, and the measurements varied erratically over this frequency range from about 0.18 pf to 0.28 pf.
An alternative is to calculate the capacitance and that of a two wire transmission-line is given by C= $\pi\epsilon_0 l_w$ / [Ln (d/r)]. For l_w =12 mm, d= 4.5mm and r = 0.5 mm this gives C = 0.152 pf. However this equation assumes that l_w >>d and so does not account for any end effect.
The acid test here is to measure the change in SRF when the leads are introduced and this can then be translated into an increase in capacitance from the equations here. In a test with an 11 turn coil on an FT 50-61 core the SRF reduced from 59.4 MHz with the minimal leads of 3 mm each, to 53.76 MHz when the leads were extended by 12mm with a gap of 4.5mm. For this change in SRF the capacitance of the extension was calculated to be 0.169 pf, which is 11% greater than that calculated from the transmission line equation.
So the most reliable estimate of lead capacitance is given by adding 11% to the transmission-line equation to give :

$$C_{leads} \approx 1.1\, \pi\epsilon_0\, l_w \,/\, [\mathrm{Ln}\,(d/r)] \quad \text{farads} \qquad 9.3.1$$

This is more conveniently given as follows for the wire length in mm and the capacitance in pf :

$$C_{leads} \approx 0.03\, l_w \,/\, [\mathrm{Ln}\,(d/r)] \quad \text{pf} \qquad 9.3.2$$

10. MORE ACCURATE MODELLING OF INDUCTANCE, RESISTANCE & Q

The self-capacitance model is very useful but the following equations give better accuracy especially at frequencies around the resonant frequency itself. The equations come from an the analysis of series and parallel resonant circuits made by Welsby (ref 7).
If a coil with a parallel capacitance is used in a parallel tuned circuit the effect of this capacitance (whether self-capacitance or added capacitance) is merely to reduce the capacitance necessary to resonate at any particular frequency. But if the coil is used in a series tuned circuit the effect of the parallel capacitance is to reduce the Q, increase the *apparent* inductance and to increase the *apparent* series resistance. Welsby derives the following equations for the series circuit, where Q, L and R, are values which would obtain in the absence of the SRF and Q_m, L_m and R_m are the values that would be measured at a frequency f, for a self-resonant frequency of f_r :

$$L_m = L\,[1 - (1+1/Q^2)\,(f/f_r)^2] \;/\; [1 - 2(f/f_r)^2 + \{(1+1/Q^2)\,(f/f_r)^4\}] \qquad 10.1.1$$

$$R_m = R\,/[1 - 2(f/f_r)^2 + \{(1+1/Q^2)\,(f/f_r)^4]] \qquad 10.1.2$$

$$Q_m = Q\,[1 - (f/f_r)^2\,(1+1/Q^2)] \qquad 10.1.3$$

If the Q is large such that $(1+1/Q^2)$] can be taken as unity, these equations simplify to :

$$\mathbf{L_m = L\,/\,[\,1 - (f\,/\,f_r\,)^2]} \qquad \mathbf{10.1.4}$$

$$\mathbf{R_m = R\,/\,[\,1 - (f\,/\,f_r\,)^2]^2} \qquad \mathbf{10.1.5}$$

$$\mathbf{Q_m = Q\,[\,1 - (f\,/\,f_r\,)^2]} \qquad \mathbf{10.1.6}$$

Although these equations were derived for lumped components they give very good predictions for the ferrite toroid, despite this having been shown to be a transmission-line. Their accuracy can be judged by calculating the inductance from Equation 10.1.1, of 10 turns wound onto a Fair-Rite Toroid 5961001801,

Material 61 and comparing with measurements. At each frequency the value of L and Q was calculated from the ferrite data at that frequency ($Q_f = \mu'/\mu''$). f_r was calculated at each frequency for the permeability at that frequency and using Equation 15.5.1.
This gave the following comparison with measurements:

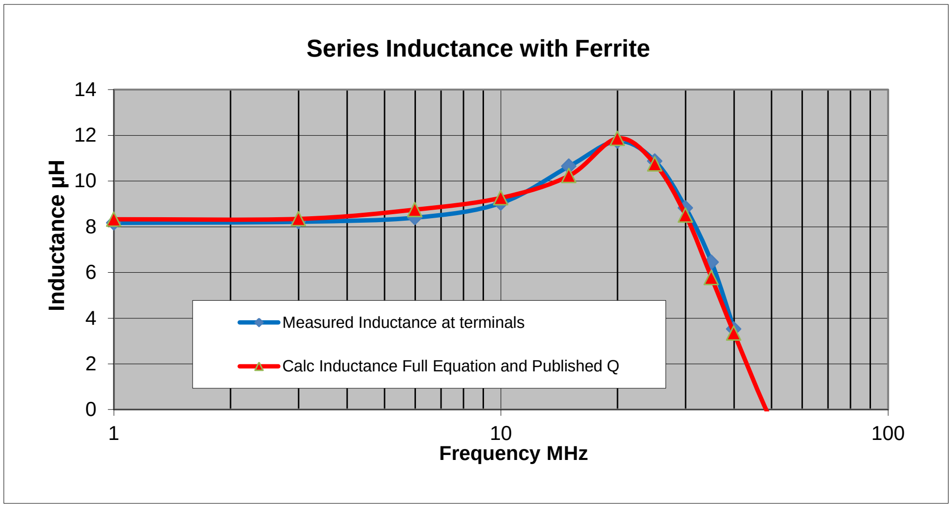

Figure 10.1 Comparison of Theory with Measurements (full equations)

The agreement is generally within ±4% which is surprising good given the uncertainty of the ferrite parameters as read by the author from the published data sheet, and the manufacturing tolerances.

For comparison the simplified equations, Equation 10.1.4 and Equation 4.1 give :

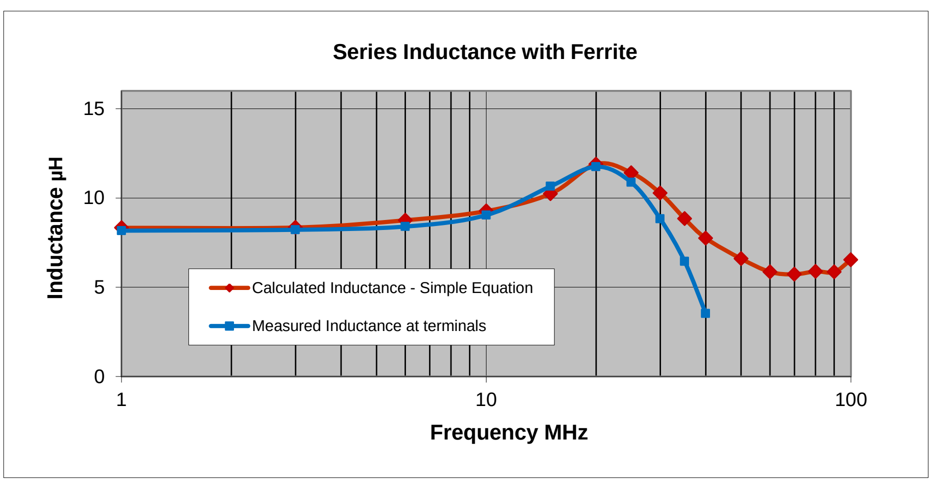

Figure 10.2 Comparison of Theory with Measurements (simple equations)

It is emphasised that for both of the above graphs f_r was calculated at each frequency, using the ferrite parameters at that frequency.

It is clear from the above that these equations are accurate enough to be a good substitute to measurements. They would also allow an analysis of the effects of manufacturing tolerances, which is difficult to do with measurements.

11. MEASUREMENTS

11.1. Test Coils

Some of the test coils are shown below.

Figure 11.1.1 Some of the test coils

From left to right these are

a) Fair-Rite 5961001801 : OD 22.1 mm, Material 61 (Ni-Zn)
b) Ferroxcube TN23/14/7 : OD 23 mm, Material 4C65 (Ni-Zn)
c) Amidon FT50-61 : OD 12.7 mm Material 61 (Ni-Zn)
d) Fair-Rite 5977000301 : OD 12.7 mm Material 77 (Mn-Zn)

Notice that the wire ends are connected to the same point on the SMA connector as the calibration loads shown below.

11.2. Test Equipment and Calibration

All measurements were made with an Array Solutions UHF Vector Network Analyser. Calibration of this analyser required an open circuit, a short circuit and known resistive load, and these are shown below.

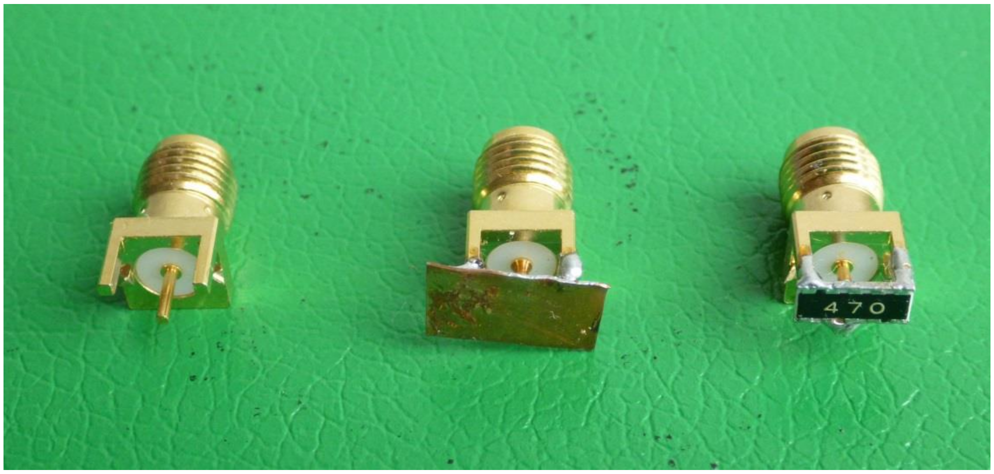

Figure 11.2.1 Calibration loads

To ensure that the resistive load had minimal stray reactance a thick-film resistor was used, and this had the added advantage that it could be located in the same plane as the short circuit. Its value was 47 Ω ± 1%. SMA connectors were used because they are small and therefore have a small stray capacitance, and so any error in calibrating this out would also be small.

11.3. Measurement Accuracy

It is difficult to assess the accuracy of the measurements since no calibrated reactances were available. However in independent tests the accuracy of the analyser compares well with the much more expensive HP Agilent 8722D (ref 11).

12. SUMMARY

The inductance of coils increases with frequency and this is often attributed to self-capacitance across the coil. However, it is shown that self-capacitance does not exist, and that the inductance increase in ferrite cored toroids is due to standing waves between the conductor and the ferrite.
Nevertheless the model of a capacitor in parallel with a fixed inductor gives a useful description of the inductance changes. It is shown here that the equation for self-capacitance has the form $C'_{self} = A + B/N^2$ and is given by :

$$C'_{self} = [1.13\ 10^{-3}\ l_e\ l_p^{\ 2}\ ] / [0.4\ \pi\ A_e)]\ +\ 10^6/[(2\pi\ k\ f_p)^2\ A_L\ N^2] \qquad \text{pf} \qquad 12.1$$

where **μ_f is the low frequency permeability of the ferrite**
l_p is periphery of the toroid cross-section in mm
l_e is the length of the magnetic path length in mm
A_L is the inductance factor given by the ferrite supplier, in $\mu H/N^2$
N is the number of turns
f_p is the frequency at which the permeability peaks in MHz
k is a ferrite material parameter, found to be equal to 2 for all ferrites investigated, both Ni-Zn and Mn-Zn.

The apparent inductance at any frequency is then given by :

$$\mathbf{L_{app} = X_t / \omega = - X_L X_C/(X_L-X_C)/ \omega} \qquad \mathbf{12.2}$$

where $\mathbf{X_L = \omega L_{fo} = \omega A_L N^2}$
$\mathbf{Xc = 1/(\omega C_{self})}$
$\mathbf{C_{self}}$ **given by Equation 12.1 or 12.4**

The manufacturer's tolerances on the ferrite parameters are quite large but it is shown that the effect on L_{app} is relatively small.
Losses due to the ferrite can be approximately modeled as a resistance in parallel with the inductor and capacitor. The value of this resistance is :

$$\mathbf{R_p = 2 \pi N^2 A_L \psi} \qquad \mathbf{12.3}$$

The factor ψ is derived from the published curves and is the product of the ferrite Q (= μ'/μ'') and the frequency at which this applies. So for instance for the Fair-Rite material 61, ψ = f (μ'/μ'') = 660 (ie at 10 MHz, μ'/μ'' = 66).
The conductor losses are generally much lower than the ferrite losses and can be ignored. The exception is when the frequency is low enough for the ferrite Q to be very high (eg several hundred), but even then the overall Q is still high even with the conductor loss.

An alternative form of Equation 12.1 which is possibly generic for all modern ferrites of all makes is :

$$\mathbf{C'_{self} \approx [1.13\ 10^{-3}\ l_e\ l_p^{\ 2}\ ] / [0.4\ \pi\ A_e)]\ +\ 10^9\ \mu_f^{\ (2x-1)}\ l_e\ / [\ 49.6\ k^2\ K^2\ A_e\ N^2]} \qquad \mathbf{12.4}$$

$\mathbf{l_e}$ **is the length of the magnetic path length in mm**
$\mathbf{l_p}$ **is the length around the perimeter of the ferrite cross-section in mm**
$\mathbf{A_e)}$ **is the area of the ferrite cross-section in** $\mathbf{mm^2}$
$\mathbf{\mu_f}$ **is the low frequency permeability of the ferrite**

Notice that the first term of this equation is dependent only upon the toroid dimensions and is independent of permeability or number of turns. In the second term the factors x, k and K have been determined for Fair-Rite materials as x=1.16, k=2 and K =5300. However it is possible that all modern ferrites have similar values and as such the above equation may be generic, requiring only the ferrite dimensions, the permeability and the number of turns.

13. APPENDIX 1 : The Measurement of Amidon FT 50-61 Permeability

To measure the complex permeability of the Amidon FT50-61 toroid it was wound with 4 turns of 0.71 mm enamelled copper wire. This small number of turns ensured that the SRF was reasonably high (> 175 MHz) so that errors in compensating for this were minimised.
An excel spread-sheet was set-up using Equations 10.1.1 and 10.1.3 to calculate the inductance and Q at each frequency, since these equations had been shown to be accurate (see Figure 10.2). The values of μ' and μ'' were then adjusted at each frequency to give the best match with the measurements (the starting values of μ' and μ'' were those of Fair-Rite 61 material).

14. APPENDIX 2 : The Effect of the Ferrite Dielectric Constant

The dielectric constant of the ferrite will reduce the phase velocity, and thereby reduce the SRF. However, the electric field inside a coil is very small, so displacement currents are very small and the effect of the dielectric is therefore much reduced. Support for this view comes from Sichak (ref 8) who has analysed a coaxial cable with a helical inner line, and who says 'The significant parameter is $(2\pi a/N)(2\pi a/\lambda)$, where N= number of turns per unit length, a=radius and λ=wavelength. When this parameter is considerably less than 1, the velocity and characteristic impedance depend only on the dimensions. The *dielectric inside the helix has only a second order effect*,………' (my italics).

The Sichak criterion $F_{Sichak} = (2\pi a/p)(2\pi a/\lambda)$ is more conveniently expressed as $4\pi\ A/(p\ \lambda)$ where p is the pitch of the winding and A the cross-sectional area. Based on Sichak's paper the author has derived the following approximate equation for the effective dielectric constant of a dielectric which totally fills the inside of the coil (ref 9) :

$$\text{Effective } \varepsilon_{eff} \approx [(\varepsilon_r - 1)\ (F_{Sichak})^{1.5}\ ]/8 \ + 1 \qquad 14.1$$

where $F_{ishcak} = (2\pi a/p)(2\pi a/\lambda_g) = 4\pi\ A/(p\ \lambda_g)$
$\lambda_g = 300/[f\ \mu_f^{\ 0.5}]$, f in MHz
A is the cross-sectional area of the ferrite
p is the pitch of the winding
ε_r is the dielectric constant of the ferrite

The above equation is valid for values of F_{Sichak} up to 1.25.

The factor F_{ishcak} increases with frequency, and taking for example 30 MHz, its value is 0.14 for the coil shown in Figure 11.1.1(a) The dielectric constant for both Ni-Zn and Mn-Zn ferrites is about 10 at high frequencies (see Snelling ref 12 p127) and so the effective dielectric constant from the above equation is 1.06. The SRF is proportional to $1/\sqrt{\varepsilon'}$ and so the effect of the ferrite permittivity is to reduce the SRF by 3%. Given the large uncertainty in the value of permeability the effect of the dielectric constant can be ignored.

15. APPENDIX 3 : Self Resonant Frequency Assuming a Transmission-line

15.1. Introduction

When a structure carries a standing wave, resonance can occur when the path length of the wave l_w is equal to multiples of $\lambda_f/4$, where λ_f is the wavelength.

This leads to resonant frequencies f_r having the general form:

$$f_r = c\ /\ [\ n\ l_w\ (1+\delta)\ (\mu_e\varepsilon_e)^{0.5}\ ] \quad \text{Hz} \qquad 15.1.1$$

where c is the velocity of light = $3\ 10^8$ m/s
μ_e and ε_e are the effective relative permeability and dielectric constant of the medium
δ is the end effect (see later)
n= 2 for half wave resonance, and 4 for quarter wave resonance

For frequencies in MHz this equation is more conveniently expressed as:

$$f_r = 300\ /\ [\ n\ l_w\ (1+\delta)\ (\mu_e\varepsilon_e)^{0.5}\ ] \quad \text{MHz} \qquad 15.1.2$$

For the toroidal coil it is is therefore necessary to determine the length of the path l_w, the end effect, and the effective permeability and permittivity.

15.2. Path Length

Initially it was assumed that the wave would follow the conductor, but experiments show that the path length is somewhat shorter than this, and is consistent with it being the projection of the conductor onto the surface of the ferrite. A consequence of this is that the diameter of the *conductor* will not affect the SRF nor

therefore the self-capacity. This conclusion was also reached by Knight (ref 2) who says ' .... it was established that there was no statistically-significant wire-diameter effect'.
So if N turns are wound onto a ferrite toroid with a magnetic length of l_e and a circular cross-section of diameter d_f and then the path length l_{path} will be :

$$\text{Circular } l_{path} = [(N \pi d_f)^2 + (\alpha\, l_e)^2]^{0.5} \qquad 15.2.1$$

If the ferrite has a rectangular cross-section with a total periphery of l_p, then the path length is:

$$\text{Rectangular } l_{path} = [(N\, l_p)^2 + (\alpha\, l_e)^2]^{0.5} \qquad 15.2.2$$

The factor α is the fraction of the toroid circumference covered by the winding. Here it is assumed that there is a 10% gap and so α =0.9 (see Section 8.3).

NB the above is based on the wave following the diameter of the ferrite. However an alternative explanation is that the wave follows the wire but the permeability is reduced by the ratio of the areas of the coil and ferrite. These two approaches lead to the same equation, as the following shows for one turn of a coil of radius a_c, wound around a ferrite of radius a_f. The length of the wire is $2\pi\, a_c$ and so the factor $l_w (\mu_e)^{0.5}$ in Equation 15.1.2 becomes $2\pi\, a_c\, (\mu_e)^{0.5}$ where μ_e is the effective permeability of the ferrite, and this is equal to $(a_f/a_c)^2\, \mu_f$, where μ_f is the ferrite permeability (see Snelling ref 12).
So we have: $2\pi\, a_c\, (\mu_e)^{0.5} = 2\pi\, a_c\, [(a_f/a_c)^2\, \mu_f]^{0.5} = 2\pi\, a_c\, (a_f/a_c)\, [\mu_f]^{0.5} = 2\pi\, a_f\, [\mu_f]^{0.5}$. So a wave following the wire radius with a permeability of $[(a_f/a_c)^2\, \mu_f$ is equivalent to assuming that the wave follows the ferrite radius with a permeability of μ_f.

15.3. Phase Velocity

For a conductor totally immersed in a ferrite material with a permeability μ and dielectric constant ϵ the phase velocity of an EM wave will be slower than that in air by the factor $\sqrt{(\mu\, \varepsilon)}$.
In general both the permeability and permittivity have loss and expressed in complex form they become:

$$\mu = \mu' - j\, \mu'' \qquad 15.3.1$$

$$\epsilon_f = \epsilon' - j\, \epsilon'' \qquad 15.3.2$$

μ' is the 'real' part of the permeability, and is that which is normally quoted as the permeability of a ferrite. μ'' is the 'imaginary' part and represents the loss. Note that the convention here is the opposite to normal circuit theory where the loss is the 'real' part. The parameters μ' and μ'' are a function of frequency and often given graphically as follows :

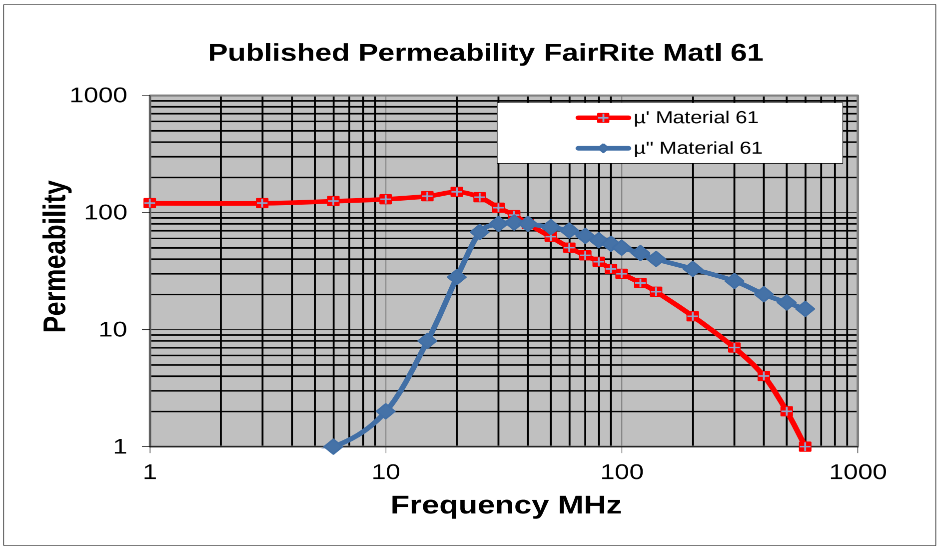

Figure 15.3.1 Typical Permeability Curves

The dielectric constant is not normally given for ferrites but Snelling gives a value of $\varepsilon' = 10$ for both Ni-Zn and Mn-Zn materials at high frequencies. The dielectric loss factor ε'' is not known.
When the conductor is wound around a toroid the conductor is clearly not immersed in the material as assumed above, but experiment shows that the phase velocity is nevertheless reduced by $\sqrt{\mu}$. This is somewhat surprising but is consistent with the conductor being a transmission-line whose inductance per unit length has increased by the same amount as the coil, that is by μ. However the same is not true for the dielectric constant, and its effect is considerably reduced because the electric field within the ferrite is very small. So it is the *effective* dielectric constant ε_{eff} which is relevant here (see Appendix 2).

When the ferrite is operated at a low frequency the loss factors have a small effect on the phase velocity, and can be ignored. However at higher frequencies μ'' can be as large or larger than μ', and so loss cannot be ignored. In this situation Skutt (ref 10) shows that the velocity is reduced by the factor :

$$\{ |\mu| |\epsilon| + (\mu' \epsilon_{eff}' - \mu'' \epsilon_{eff}'') \}^{0.5} / 2^{0.5} \qquad 15.3.3$$

NB In Figure 15.3.1 the real part μ' is shown heading for a value of less than unity, and this is most unlikely. The curve is probably that of the magnetic susceptibility, (μ'-1), especially since the standard coaxial measurement method naturally gives this rather than μ' (see Hamilton ref 3).

15.4. End Effect

A transmission-line will often appear to have a greater length than its physical length because the fields extend beyond its ends. This is the so-called end-effect which is common in all open ended resonators, including transmission-lines and organ pipes. However in the toroid the flux is totally contained and no end-effect has been found.

15.5. Equation for SRF

Putting all the above together gives the following SRF for λ/2 resonance :

$$f_{\lambda/4} = 300 / [2\, l_{path} \; \{ |\mu| \, |\epsilon_{eff}| + (\mu' \epsilon_{eff}' - \mu'' \epsilon_{eff}'') \}^{0.5} / 2^{0.5}] \quad \text{MHz} \qquad 15.5.1$$

where $|\mu| = [\mu'^2 + \mu''^2]^{0.5}$

$|\epsilon| = [(\epsilon_{eff}')^2 + (\epsilon_{eff}'')^2]^{0.5}$
$l_{path} = [(N\, l_p)^2 + (\alpha\, l_e)^2]^{0.5}$
N is the number of turns
l_e is the length of the magnetic path in metres
l_p is the length of the periphery of the cross section in metres
α is the proportion of ferrite with winding (ie 0.9)

This equation can be simplified if the dielectric constant is assumed to be unity and the ferrite is operated at a frequency where the ferrite Q is above unity, ie μ' > μ'' (this corresponds to 20 MHz for material 61 (Figure 4.1).

Self-resonant frequency $f_r \approx 300 / [\, 2\, l_{path}\ (\mu')^{0.5}]$ MHz **15.5.2**

16. APPENDIX 4 : Comparison with Published Measurements

It is useful to compare the theoretical values for C_{self} with the only published measurements which the author could find – those of Knight ref 2. He wound 22 coils on an Amidon FT 50-61 ferrite core (always the same core) with from 4 to 36 turns. He gives the equation for the self-capacitance of this core as :

$$C_{self} = 0.90 + 78.1/N^2 \quad \text{pf} \qquad 16.1$$

To derive this equation he ' …resonated the coil against a series of known capacitances, fitting the data to a regression line, and extrapolating to find the resonant frequency when C=0'.

For comparison Equation 7.3.1 gives :

$$C_{self} = 0.49 + 106/N^2 \quad \text{pf} \qquad 16.2$$

These equations are plotted below:

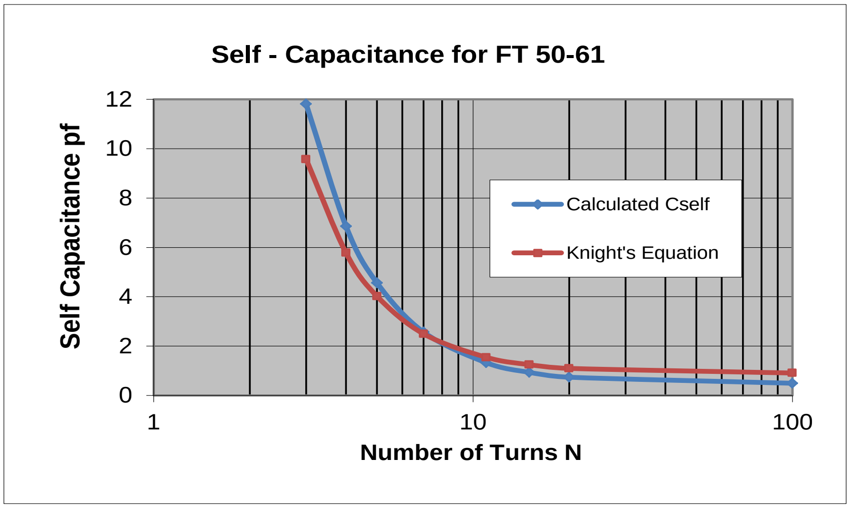

Figure 16.1 Comparison of Theory with Knight's measurements

The tolerance in the theoretical value due to manufacturing tolerances is ±12% on the first term and ±20% on the second term. So the first term could be between 0.43 and 0.59 and the second term between $84.8/N^2$ and $127/N^2$.

The largest percentage difference between Knight's measurements and the theoretical curve is when N is large. The capacitance is then very small at 0.9 pf for Knight's equation and 0.49 pf for the theoretical equation. However Knight's equation includes the capacitance of his leads, which he estimated to be 0.05 pf, but Equation 9.3.2 gives the value as 0.15 pf. Assuming this is a better estimate of his lead capacity this would bring his first term down to 0.8 pf, which is within 0.21pf of the maximum likely theoretical value of 0.59 pf, and probably within his experimental error. Interestingly Knight says at the end of his article '……..it appears that the non-electrostatic component of the self-capacitance settles to about 0.40 ±0.11 pF as the number of turns becomes very large', and this is consistent with the first term of Equation 16.2.

As for the second term his value of $78.1/N^2$ is 8% less than the minimum theoretical value. The magnitude of this term is related to the rising ferrite permeability and so it is significant that Knight says 'In the present work however, the evidence is that the hump in permeability in the 8 to 30 MHz range of the manufacturer's graph is attenuated or absent in the actual sample'. In that case one would expect a lower magnitude than that given by Equation 16.2.

17. APPENDIX 5 : Self Capacitance in Physical Dimensions Only

Equation 6.1 gives the self-capacitance when the permeability is constant, and is expressed in terms of μ_f and A_L since these are parameters normally given by the manufacturer. However μ_f and A_L are not independent and their relationship is given in Section 9.1 as $\mu_f / A_L = l_e / (\mu_0 A_e)$. Substituting this into Equation 6.1 this becomes :

$$C_{self} = 1.13\ 10^{-3}\, l_e\, l_p^{\,2}\, [1 + \{ \alpha\, l_e /(N\, l_p)\}^2] / (0.4\, \pi\, A_e) \text{ pf} \qquad 17.1$$

Where N is the number of turns
l_e is the length of the magnetic path in mm
l_p is the peripheral length of the cross-section in mm
A_e is the cross-sectional area of the ferrite in mm^2
α is the proportion of ferrite with winding (ie 0.9)

Writing this in the form $C_{self} = A + B/N^2$ gives :

$$C_{self} = [1.13\ 10^{-3}\ l_e\, l_p^{\,2}] / [0.4\, \pi\, A_e)] + 1.13\ 10^{-3}\, (l_e)^3\, \alpha^2] / [0.4\, \pi\, A_e] / N^2 \quad \text{pf} \qquad 17.2$$

Notice that the self-capacitance is dependent only upon the ferrite dimensions and the number of turns, and is independent of the permeability, or the coil diameter or the wire diameter (when the permeability is constant).

18. APPENDIX 6 : Compensation for changing Permeability, Method 2

Section 7 gives a method for compensating for the change in ferrite permeability, but it works only if the curve of rising permeability matches that of the increasing inductance of a parallel resonant circuit. Fortunately most ferrites seem to have this characteristic, but if the ferrite had, say, a linearly rising inductance this method would not work.

The alternative method is to retain Equation 6.1 unaltered (which assumes constant permeability), and to modify Equation 6.3 by making A_L frequency dependent. This frequency dependence can be tailored for any rise in permeability.

So the task here is to find an equation which reflects the change in inductance due to the permeability changes. As an example the following empirical equation does this over the useful frequency range for Material 61, and probably for most known ferrites:

$$L_{fo}' = L_{fo} / [1 - (f/f_a)^n] \qquad 18.1$$

The factors f_a and n are found by matching the above equation with the curve of μ' versus frequency for the ferrite being used. For the Amidon Material 61 a good match was found with f_a = 73 and n= 1.5. This gives:

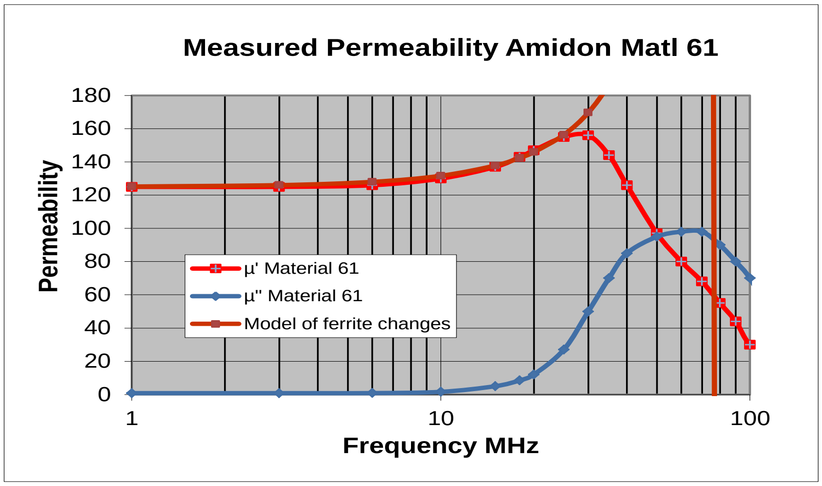

Figure 18.1 Self Capacitance Model cf Measured Inductance

Of course Equation 18.1 is only one of many possible equations which can be found to match the changing permeability with frequency. However, continuing with this example Equation 18.1 can also be expressed as :

$$A_L' = A_L / [1 - (f/f_a)^n] \qquad 18.2$$

It would be useful to find generic values for f_a and n, for any ferrite. It is assumed here that n is the same for all ferrites at 1.5 and that f_a scales according the frequency at which the permeability peaks, f_p. This frequency for the Amidon material 61 is 28 MHz, so $f_a = 2.6 f_p$. So a possible generic equation for A_L' is :

$$A_L' \approx A_L / [1 - (0.38f/f_p)^{1.5}] \qquad 18.3$$

where f_p is the frequency at which the permeability peaks

The apparent inductance is now:

$$L_{app} = - X_L X_C/(X_L - X_C)/\omega \qquad 18.4$$

where $X_L = \omega A_L' N^2$
$X_c = 1/(\omega C_{res})$
C_{res} given by Equation 6.3
$A_L' \approx A_L / [1 - (0.38f/f_p)^{1.5}]$
f_p is the frequency at which the permeability peaks

If Equation 18.4 is applied to the 11 turn coil on the FT 50-61 toroid, it gives the following comparison with measurements :

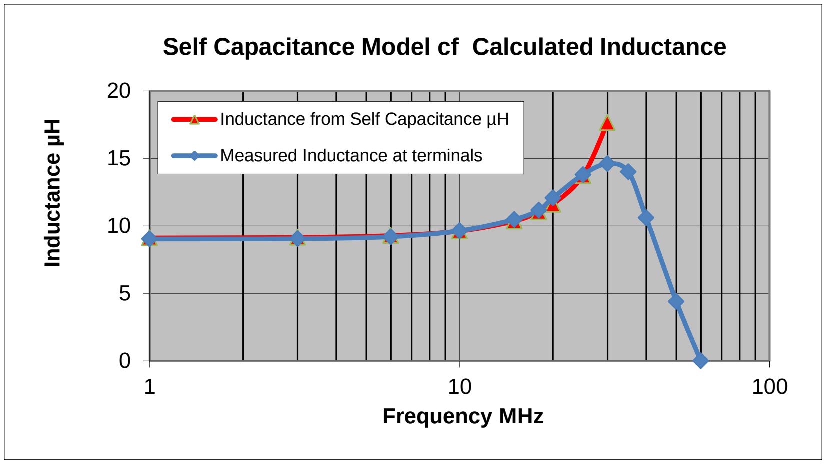

Figure 18.2 Plot of Equation 18.4 cf Measurements

19. APPENDIX 7 : Tolerances and Uncertainty

19.1. Introduction

The degree to which the equations given in this article agree with measurements is dependent upon the accuracy of the toroid parameters as supplied by the manufacturer. Some of these parameters have a large tolerance and the effect of this on the accuracy of the equations is assessed below.

19.2. Uncertainty in Permeability

The uncertainty in the value of A_L is ± 20%. Some of this is attributable to the permeability, and some is due to the tolerances on the physical dimensions. For instance for the Amidon FT50-61 the following dimensions are given : OD 12.7 mm ±0.25 mm, ID 7.15 mm ±0.12 mm and height 4.9±0.25 mm. So the % tolerance on each is: OD ±2%, ID ±2%, and height ±5%. If the mean length of the magnetic path l_e has the same uncertainty as the OD and ID this also has a tolerance of ±2%.
If all these tolerances are uncorrelated we can assume they add in quadrature, so that if the uncertainty on the permeability is x, then $[(2^2+2^2+5^2) + 2^2) + x^2]^{0.5} = 20$ (the first three factors give the uncertainty in the area, and the fourth the uncertainty in the magnetic path).
The uncertainty on permeability, x, from this equation is ±19 %, so the effect of the physical uncertainties on A_L are very small.
This uncertainty in permeability is reflected in the same uncertainty in the frequency at which the permeability peaks f_p, and thus the start of the roll-off. This is because the product of permeability and the -3dB roll-off point are approximately constant for any particular material, and this is known as the Snoek product (see Figure 7.5.1). So a lower permeability ferrite will have a higher roll-off frequency and vice-versa, and thus we can expect an uncertainty on f_p of ±19 %.
There is also a difference between manufacturers for what they claim is the same material, as shown below for material 61:

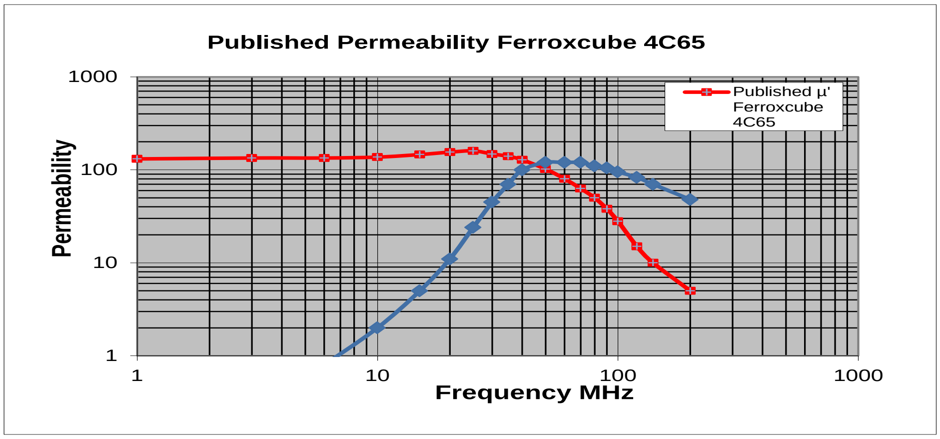

Figure 19.2.1 Ferroxcube 4C65

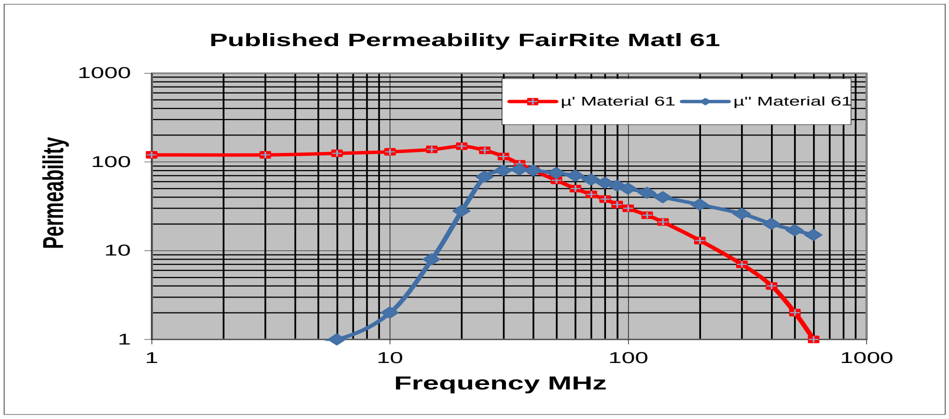

Figure 19.2.2 FairRite Material 61

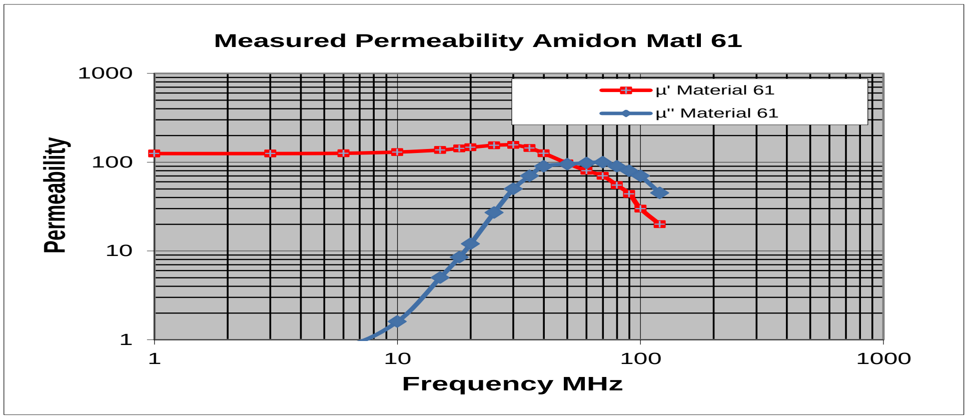

Figure 19.2.3 Amidon Material 61(Author's measurements)

As an example of the difference, the frequency at which the permeability peaks is 25 MHz for Ferroxcube, 20 MHz for Fair-Rite and 30MHz for Amidon, a range of ± 25%.
There is some doubt about the accuracy of Ferroxcube curves because a core TN23/14/7 having material 4C65, and wound with 10 turns gave an inductance v frequency which did not correspond with the Figure 19.2.1 but agreed almost exactly with the FairRite data Figure 19.2.2.
As for Amidon, the author has been unable to find any published permeability information from them, and so the data in Figure 19.2.3 is from the author's own measurements.
In contrast Fair-Rite publish full data on all their materials, and in the author's experience this is representative of the purchased item.

19.3. Uncertainty in Physical dimensions

The % tolerance on the linear dimensions are OD ±2%, ID ±2%, height ±5%. If the mean length of the magnetic path l_e has the same uncertainty as the OD and ID this is also ±2%.
The width of the toroid is (OD-ID)/2 and so the cross-sectional area A_e will have an uncertainty of $[(2^2+2^2+5^2)]^{0.5}$ and this is equal to ±6%.
The peripheral length around the cross-section l_p will have an uncertainty of $[(2^2+2^2+5^2)]^{0.5}$ = ±6% (assuming the OD and ID tolerances are uncorellated).

19.4. Uncertainty in Derived parameters

The factors f_p , k, K and x are derived from the published permeability curves.
For f_p there is an uncertainty in determining the peak permeability from the published curves of say ±5%, and this is to be added to the manufacturing tolerance of ±19% (the same as permeability) giving a total of $[(19^2+ 5^2)]^{0.5}$ = ±20 %.
For K and x, these have only been determined for the Fair-Rite ferrites, and it is assumed that the combined uncertainty is the same as for f_p, at ±19 %.
For k, its value of 2 appears to be universal for all modern ferrites, and it is assumed to have an uncertainty of ±5%.

19.5. Uncertainty in Equation 7.3.1

If we express the self-capacitance as $C'_{self} = A + B/N^2$, then the uncertainty in A and B is estimated as follows: The factor A has uncertainty in μ divided by A_L, but these are correlated and so the errors cancel. The square of the periphery l_p will have an uncertainty of 2 * 6 = ±12 %.

The factor B has uncertainty in f_p^2/A_L, and given that the $f_p \propto 1/\mu$ and $A_L \propto \mu$ the overall uncertainty is ±20%.

19.6. Uncertainty in Equation 7.5.2

If we express the self-capacitance as $C'_{self} = A + B/N^2$, then the uncertainty in A and B is estimated as follows.
The factor A has uncertainty in $l_e\, l_p^{\,2} / A_e$. There is some correlation between the lengths and the area, and so a tendency for the errors to cancel, so it is estimated that the combined uncertainty is $[(2^2+12^2-6^2)]^{0.5}$ = ±11%.

The factor B has uncertainty in $\mu_f^{\,1.16}\, l_e \,/\, [\,k^2\, K^2\, A_e]$ giving a total of $[(19^2+2^2+5^2 +19^2 +19^2 + 6^2)]^{0.5}$ = ±35%.

20. APPENDIX 8 : Simplification of Equation 7.2.2

The middle term of Equation 7.2.2 is often much smaller than the sum of the other two terms and can therefore be ignored, as can be seen from Equation 7.2.3.
The error in ignoring this term depends upon the number of turns N, the permeability μ, and the frequency at which the permeability peaks f_p. The error is also dependent upon the toroid geometry in that this determines the ratio of l_e^2 / l_p^2. This has a surprisingly large effect because, in commercially available

toroids, as the overall diameter increases the relative cross-section decreases (ie the toroid becomes slimmer). For example the following curves show the error in discounting the middle term for 5 turns wound onto toroids with an overall diameter of 12.7, 22.1, 35 and 61 mm :

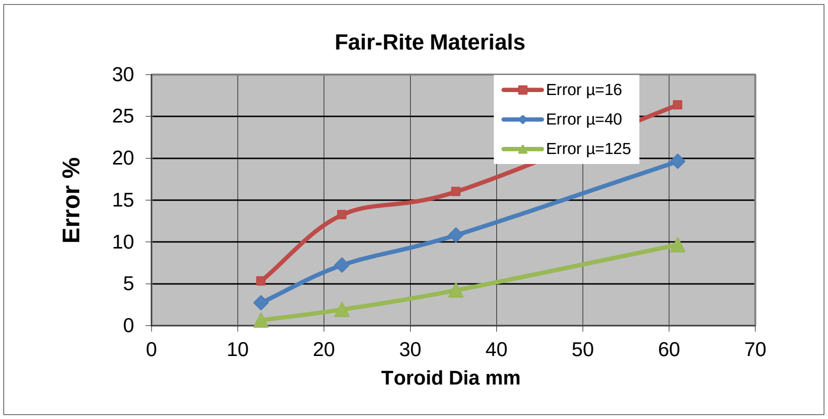

Figure 20.1 Error for N=5, irrespective of core diameter

However this comparison is unrealistic in that it assumes that the large toroids have the same number of turns as the smallest : whereas it is reasonable to consider 5 turns on the 12.7 mm toroid this small number is most unlikely on a 61 mm toroid. More realistic is to assume that the number of turns increase in proportion to the diameter, albeit with a small number of turns since this is the worst case. If the number of turns is : 12.7 mm 5 turns, 22.2 mm 9 turns, 35 mm 14 turns, and 61 mm 24 turns, the error in discounting the middle term becomes :

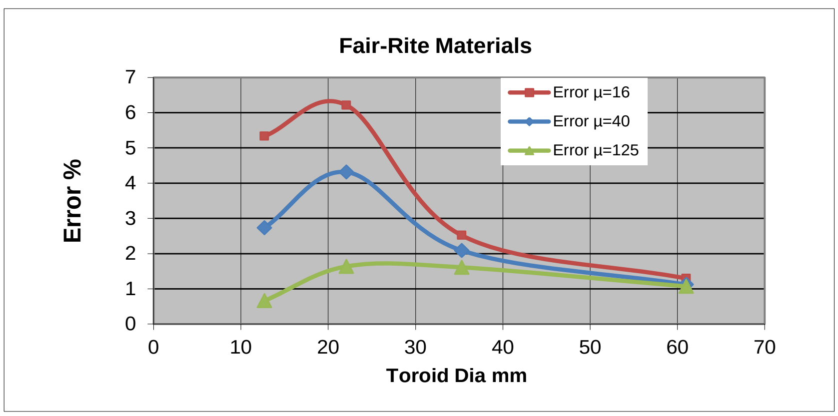

Figure 20.2 Error for N proportional to diameter

This shows that the error in discounting the middle term is less than 6% if the permeability is not less than 16 and the number of turns is not unrealistically small for the diameter of the ferrite.

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