THE INDUCTANCE OF FERRITE ROD ANTENNAS

When a ferrite rod is inserted into an air coil its inductance increases by a large factor, but the widely quoted equations for predicting the new inductance are shown to be flawed. A new theory is presented, based upon the magnetic reluctance, and this gives accurate predictions compared to experiment. Interestingly this shows that the increase in inductance when the ferrite is introduced is independent of the number of turns or their spacing or the inductance of the original air coil. Also if the ferrite permeability is high the increase is dependent only on the overall physical dimensions of the coil and ferrite.

1. INTRODUCTION

Ferrite rods have been used as slugs in air coils in order to increase their inductance and to provide a convenient method for adjusting the inductance. They have also been used in ferrite-rod antennas, to increase the radiation resistance of the coil (Payne ref 1). In both cases it is important to be able to calculate the inductance when the rod is inserted, but the widely quoted equations are only approximations. This poor situation is recognised by many authorities and for instance Lo and Lee (ref 2 p6-22) say 'It is noted that no satisfactory formulas for the inductance of ferrite antennae are available', and Miron (ref 3) says 'there is no analytical help in determining inductance'

Some of the ideas presented here were first published by the author in reference 4. However a number of the equations there were based upon measurements, and these assumed that the permeability of the ferrite rods were as quoted by the manufacturer. It is now realised that the actual permeability of ferrite *rods* is generally much lower than quoted, by a factor of 2 or more, and so some of these equations are not accurate. This problem is addressed here and more accurate equations for the calculation of inductance are presented.

In the following text, key equations are highlighted in red.

2. CONVENTIONAL DEMAGNETISATION EQUATION

If a ferrite rod with a permeability of say 100 is inserted into an air coil the inductance increases, but not by a factor of 100, but by a much lower factor (see later). So the ferrite seems to have a reduced permeability, designated μ_{rod}, which is then used in the conventional theory to determine the inductance with the ferrite, L_f. For instance the 'Antenna Engineering Handbook', Johnson & Jasik (ref 5, Chapter 5) gives:

$$L_f = \mu_{rod}\, F_L\, \mu_o N^2 A / lc \qquad 2.1$$

Where N is the number of turns, A is the cross-sectional area of the coil and *lc* its length. For the calculation of μ_{rod} the theory of demagnetisation is used, borrowed from the theory of permanent magnets (see Snelling ref 6 p182). However equation 2.1 gives values for inductance which are not very accurate, and F_L is an empirical factor needed to get the equations to agree with experiment. This factor ranges from 0.2 to 0.7, but even then is not accurate because F_L is derived from 'averages of experimental data' (ref 5).

3. THE CHANGE IN INDUCTANCE WHEN A CORE IS INSERTED

In contrast to the demagnetisation theory above, the approach used here makes use of the extensive theory for the inductance of air-cored coils. So rather than establishing the absolute value of the inductance when the core is introduced (ie in terms of N^2, A l as above), the theory developed here calculates the *change* in inductance when the core is inserted. This has the advantage that the current body of knowledge for air coils is retained and used to establish the basic air inductance. The new theory now only has to determine the *increase* when the core is introduced. So the theory which follows is in terms of the ratio L_f/L_{air}, where L_{air} is the inductance of the air coil and L_f is the inductance after the ferrite has been inserted.
So in terms of the ratio L_f/L_{air}, the inductance with ferrite is equal to :

$$L_f = L_{air}\ (L_f/L_{air}) \qquad 3.1$$

The ratio (L_f/L_{air}) is commonly called μ_{rod}, but this designation is avoided here to avoid confusion with the permeability of the rod *material*.

A summary of equations for an air coil is given in Appendix 1.

4. RELUCTANCE INSIDE AND OUTSIDE A COIL

Magnetic flux flows down the inside of a coil, exits at one end, and flows externally to enter the coil at the other end. The coil therefore can be considered as having two magnetic paths, that inside the winding and that outside, and the effect of inserting a ferrite core is dependent upon the relative reluctance of these two paths (for an introduction to magnetic reluctance see ref 7). So if the inside reluctance is larger than the outside reluctance then the insertion of the ferrite will have a large effect. Conversely if the inside reluctance is small then the ferrite will have only a small effect. So this ratio of the inside reluctance to the outside reluctance is fundamental to the effect of a ferrite core, and this ratio is given by (see Appendix 2) :

$$\mathcal{R}_{in\ air} / \mathcal{R}_{out\ air} = 5.1\ [l' / d_c]/[1 + 2.8\ (d_c/ l')] \qquad 4.1$$

$$\text{where } l' = l_c + 0.45\ d_c$$

This equation has been derived from experiments using ferrite cores (and one theoretical point, see Appendix 2), but is believed to also apply to air coils. It has an estimated accuracy of ± 4.5%, for l_c / d_c ratios from 0.5 to ∞. Equation 4.1 is plotted below:

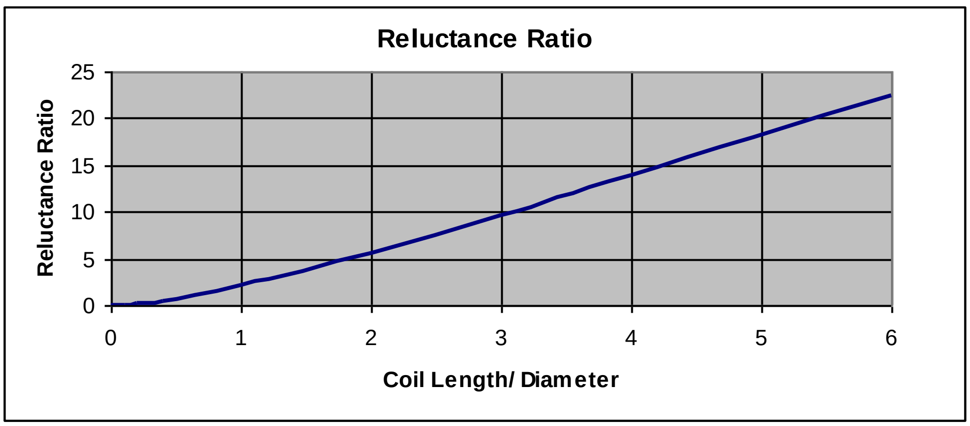


Figure 4.1 The Reluctance Ratio of an Air Coil

Taking a typical air coil with a length twice its diameter, this has an inside reluctance about 6 times the outside reluctance, and so ferrite inside the coil will have a much greater effect than ferrite outside. If the coil length is half the diameter then the inner and outer reluctances are about equal, and the effect of ferrite

is about equal. If the coil is very short compared to its diameter, the outside reluctance is much larger than the inside reluctance, and ferrite inside the coil will have little effect on the inductance compared with ferrite outside.
Notice that if the coil is very long and thin, the inside reluctance totally dominates and no significant increase in inductance is achieved by extending the ferrite beyond the ends of the coil (although there is an advantage in reducing the loss).
The coil diameter d_c in the above is the diameter at which the current flows, and for an air coil wound with Litz wire the current will flow down the centre of the wire on average. In contrast, for solid wire at high frequencies the current will tend to flow on the inside of the wire closest to the axis of the coil. So d_c for solid wire is assumed here to be the inner diameter of the wire ie equal to the diameter of its winding former.
Equation 4.1 can be used to calculate the increase in inductance when the ferrite is introduced, L_f/L_{air}, and this is the topic of the next section.

5. EQUATION FOR L_f/L_{air}

5.1. The Inductance Ratio

The ratio of the inductance with and without the ferrite core is given by the following equation (see Appendix 3):

$$L_f/L_{air} = (1 + x) \;/(1/k + x/ \mu_f) \qquad 5.1.1$$

Where x is given by Equation 4.1

$1/k = \mathcal{R}_{out\ f} \;/\; \mathcal{R}_{out\ air}$

μ_f is the relative permeability of the ferrite

The numerator (1+x) is the sum of the outside reluctance and the inside reluctance, both normalised to the outside reluctance, for the air coil. The denominator is the same but with the ferrite present, and here the inside reluctance x is seen to be reduced by the relative permeability of the ferrite μ_f. The outside reluctance is reduced by the factor k, and so k is the apparent relative permeability of the outside flux path due to the ferrite extending beyond the ends of the coil (see Section 7).

5.2. Apparent Permeability

The above assumes that the ferrite inside the coil totally fills the space, but in practice the ferrite radius must be slightly smaller than the coil radius, leaving a small radial gap. If this gap is small the magnetic susceptibility of the core is reduced by the ratio of the areas $(a_f /a_c)^2$ giving :

$$\mu_{fe} = (\mu_f - 1)\,(a_f /a_c)^2 + 1 \qquad 5.2.1$$

When the ferrite permeability is much larger than unity the above equation reduces to $\mu_{fe} = \mu_f\,(a_f /a_c)^2$.

5.3. The Permeability of Ferrite Rods

The permeability of ferrite *rods* is very difficult to measure, and so manufacturers usually quote the permeability which they have measured using a toroid of the same material. However the forming pressures are often very different and it is the author's experience that in reality the rod permeability is only half or less that of the toroid. Evidence for this is given Section 6.

6. INDUCTANCE WITH INTERNAL FERRITE ONLY

The arrangement now considered is where the ferrite fills the inside of the coil but there is no ferrite outside the coil, so the coil is exactly the same length as the ferrite and essentially the same diameter (Figure 6.1).

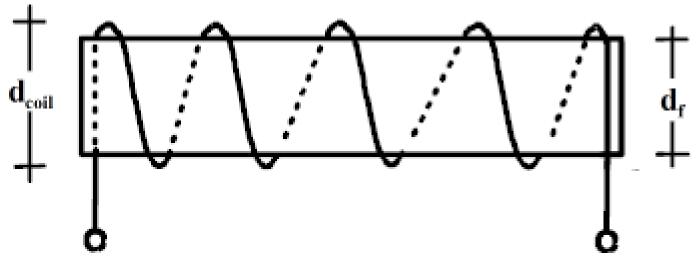


Figure 6.1 : Coil and Ferrite of Equal Lengths

With no ferrite outside k=1 and Equation 5.1.1 becomes :

$$L_f/L_{air} = (1 + x)/(1 + x/\mu_{fe}) \qquad 6.1$$

where x is given by equation 4.1
μ_{fe} is given by equation 5.2.1

This is plotted below for various values of effective permeability up to infinity:

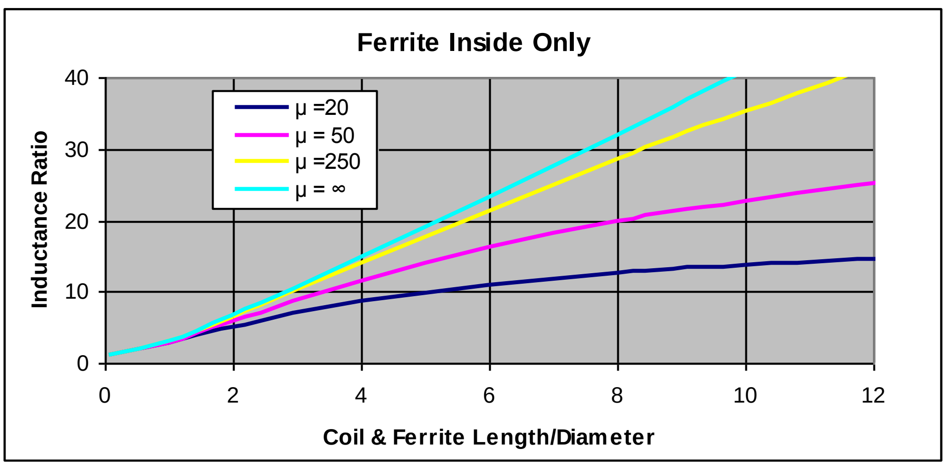


Figure 6.2 : Inductance ratio for Coil with Ferrite Inside only.

Notice that when the permeability is very high, so that x/μ_{fe} is very much less than unity, then $L_f/L_{air} \approx (1+x)$.

A comparison of Equation 6.1 with experiment is shown below :

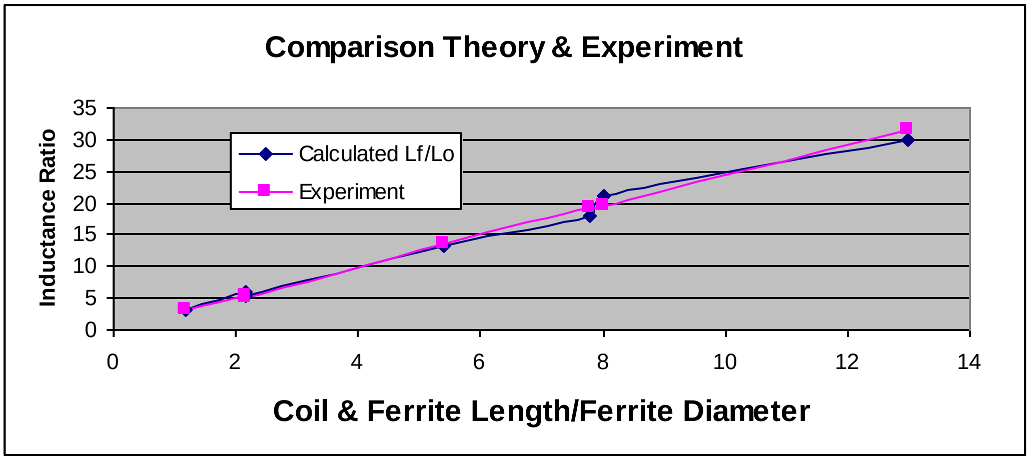


Figure 6.3 : Inductance Ratio

For these measurements the ferrite diameter was 9.25mm, and cut to the lengths as indicated in the graph above. For each ferrite length a coil was close wound onto the ferrite along its whole length, sometimes directly onto the ferrite using wire of 1.6mm dia or 0.5mm dia, and sometimes with stranded wire of 0.23mm dia onto formers with an outside diameter of between 10.1 and 11.2mm. The number of turns ranged from 7.5 to 225.

This convincingly shows that the ratio L_f/L_{air} is independent of the number of turns, the diameter of the winding, the diameter of the wire and the inductance of the air coil, and dependent only on the physical dimensions of the coil and ferrite (for this fixed value of permeability).

[NB The permeability of the ferrite rod was given as 325 by the suppliers, but a value of 90 was used for the theoretical curve above, since this gave the best match to the measurements. Indeed any one of the above measurements provided a method for determining the rod permeability, from Equation 6.1].

7. THE INDUCTANCE WHEN THE FERRITE EXTENDS BEYOND THE END OF THE COIL

The case is now considered where the ferrite extends beyond the ends of the coil, and is centred on it, see Figure 2.

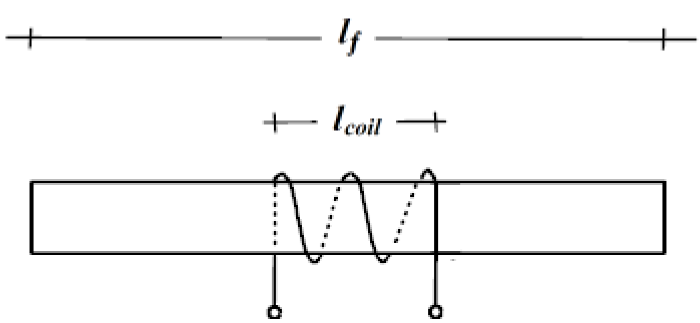


Figure 7.1 : Ferrite Longer than Coil and Centred.

It is assumed that ferrite which extends beyond the end of the coil has no effect on the reluctance inside the coil, but it does reduce the reluctance outside.

To determine the factor k (the apparent permeability of the outside path), an equation is needed describing the magnetic field around the extended ferrite. No such equation is known but often magnetic fields have the same shape as electric fields, and so equations for *capacitance* can be used to solve magnetic problems (see Payne ref 7). The following relationship can therefore be used for the ratio of the reluctance of the external path without the ferrite to that with ferrite :

$$\mathcal{R}_{\text{out air}} / \mathcal{R}_{\text{out f}} = k = C_{\text{out f}} / C_{\text{out air}} \qquad 7.1$$

$C_{\text{out f}}$ is the capacitance between the two arms of the ferrite protruding beyond the ends of coil, and $C_{\text{out air}}$ is the *external* capacitance between two discs having the same radius as the coil. So the ratio between these two gives the increase in external capacitance when the ferrite is introduced, and thereby the reduction in the reluctance of the external flux path.

$C_{\text{out air}}$ is dependent on the spacing of the discs ie the length of the coil, but this dependency is weak for normal lengths and so it can be approximated to the capacitance of a disc, with only one face (Schelkunoff & Friis ref 11 p244):

$$\mathbf{C_{\text{out air}} \approx 4\, \varepsilon_0\, a_c} \qquad \mathbf{7.2}$$

where $\mathbf{a_c}$ is the radius of the coil

For $C_{\text{out f}}$ it is assumed that the magnetic field is similar to the *electric* field around a normal electric dipole antenna, and this will consist of two components, that due to the sides of the protruding ferrite $C_{\text{an f}}$ and that due to its ends $C_{\text{end f}}$. The latter due to the ends is the same as Equation 7.2 above but for the ferrite radius :

$$\mathbf{C_{\text{end f}} \approx 4\, \varepsilon_0\, a_f} \qquad \mathbf{7.3}$$

where $\mathbf{a_c}$ is the radius of the ferrite

For that due to the sides $C_{\text{an f}}$, Schekunoff & Friis (ref 11) give the capacitance of a dipole antenna as:

$$C_{\text{an f}} = [\, \pi\, \varepsilon_0\, h / (\, \text{Ln}\, 2\, h/a \; -1)\,] \qquad 7.4$$

Where h is the half length of the dipole
a is its radius

In applying this equation here there are two considerations. Firstly, the interest here is only that part of the ferrite which extends beyond the end of the coil, so h becomes $h = (h_f - h_c)$, where these are the half lengths of the ferrite and coil respectively. Secondly, Equation 7.4 is only accurate for long thin cylinders whereas the ferrite rod is likely to be much fatter. Capacitance equations for this situation are unnecessarily complex for this application, and the following empirical equation has agreed well with experiments :

$$\mathbf{C_{anf} = 0.5\, \pi\, \varepsilon_0\, (\, \mathit{l}_f - \mathit{l}_c\,) / [\, \text{Ln}\, \{2\, (\, \mathit{l}_f + d_f) / d_f\, \} \; - \; 1\,]} \qquad \mathbf{7.5}$$

where $\mathbf{\mathit{l}_f}$ **is the length of the ferrite rod and** $\mathbf{d_f}$ **its diameter**
$\mathbf{\mathit{l}_c}$ **is the length of the coil**

So the factor k, Equation 7.1, is equal to :

$$k = [\, C_{anf} / \varepsilon_0 + 2\, d_f\,] / \; 2\, d_c \qquad 7.6$$

where C_{anf} is given by equation 7.5

Equation 4.1 with 5.1.1 and 7.6 give the following curve for L_f/L_o , for various lengths of ferrite.

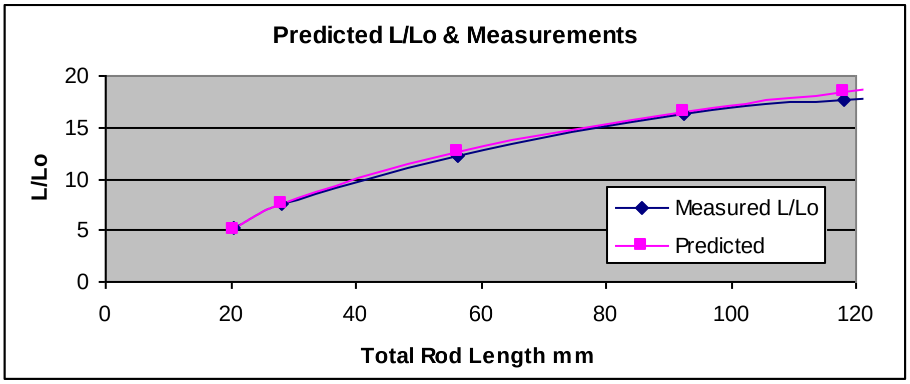


Figure 7.1 Comparison with Experiment

This agrees with the measured values to within ±5%. The error is greatest at the longest rod length and increases as the rod is made even longer, and the reason for this is discussed in the next section.
The ferrite material here was the same as that for Figure 6.3, and so a permeability of 90 was used, rather than the manufacture's value of 325.

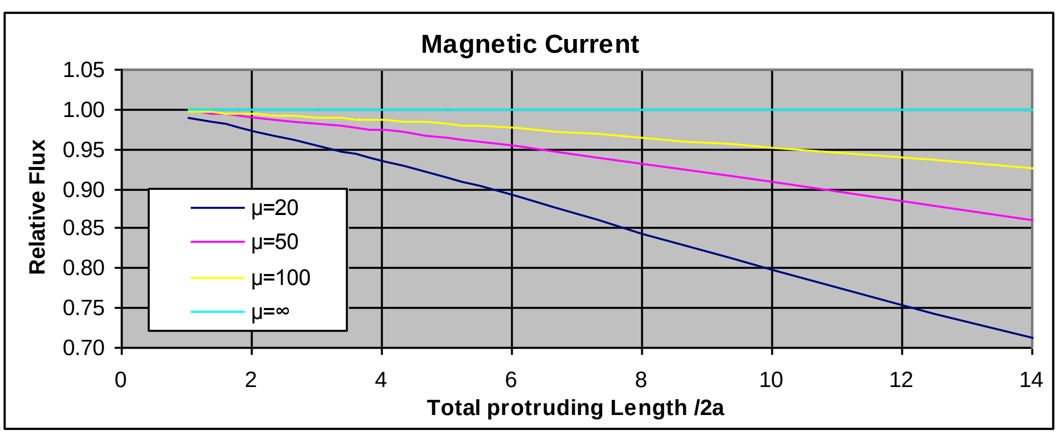
8. MAGNETIC CURRENT (Flux)

The equations for capacitance assume that the conductor has infinite conductivity, and for normal electric circuits using metals this gives a negligible error. However, the ferrite rod has a relatively poor 'conductivity' to the flux because it has a finite permeability, and so the actual magnetic current (flux) is less than it would be if the permeability was infinite. An analysis shows that the ratio of the actual flux to that with infinite permeability is approximately given by (Appendix 4):

$$\varphi / \varphi_{max} \approx 1 \Big/ \left[1 + \{(l'_f / d_f)^{1.4}\} / (5\,\mu_f)\right] \qquad \textbf{8.1}$$

where l'_f is the length of protruding ferrite = $(l_f - l_c)$

This equation is plotted below :



Normally the ferrite rod would be no longer than 12 times its diameter, and its permeability would be greater than 100, and then the reduction in flux is no more than 5%. So this effect is only significant if the rod is especially long compared with its diameter, and/or it has a low permeability.

With this factor Equation 7.6 becomes :

$$\mathbf{k = [\ (\varphi/\ \varphi_{max}\ C_{anf}\ /\ \varepsilon_0\ )\ \ + 2\ d_f\ ]\ /\ \ 2\ d_c} \qquad \mathbf{8.2}$$

where C_{anf} is given by equation 7.5
d_f and d_c are the diameters of the ferrite and the coil
$\varphi/\ \varphi_{max}$ is given by equation 8.1

9. SIMPLIFIED EQUATION

The equation for L_f/L_{air} can be simplified if the following assumptions are made :

a) The coil length is no longer than twice its diameter
b) μ_f is very large
c) The coil radius and the ferrite radius are the essentially the same,

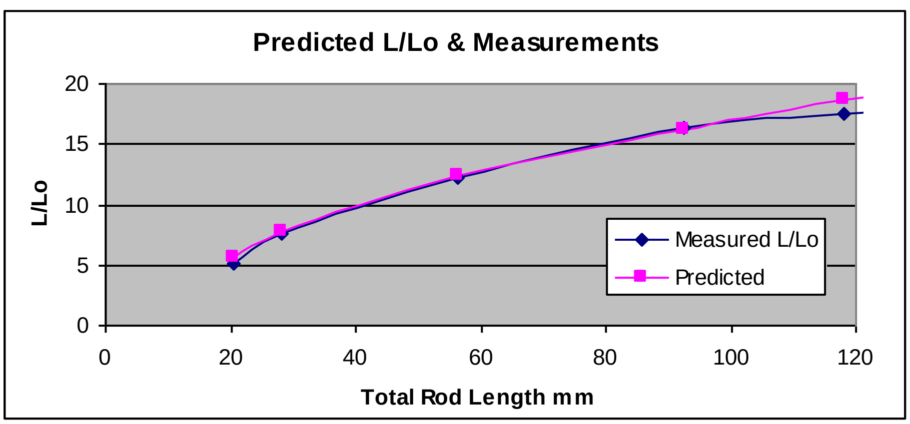
Then $L_f/L_{air} \approx$ (1+x)k [from a) and b) above] although better agreement for finite permeabilities is given if $L_f/L_{air} \approx$ xk). From c) above k≈ [C' +1], where C' = C_{anf} / $C_{out\ air}$

With these assumptions,

$$\mathbf{L_f/L_{air} \approx x\ (1+C')} \qquad \mathbf{9.1}$$

Where **x is given by equation 4.1**
$C' \approx m\ [(l_f - l_{coil}\ )\ /\ d_{coil}\ ]\ /\ [\ Ln\ \{2\ (l_f + d_f)/\ d_f\ \}\ -1]$

The multiplier m above is equal to 0.5π/(4/2) = 0.79 but better agreement with experiment is given by m=0.7. Equation 9.1 then gives, for the coil and ferrite in Figure 7.1 :



The agreement with experiment is still good with these simplifications, but the above equation has only been tested against this one coil.

10. SUMMARY

In summary, the inductance of a ferrite rod antenna of any length and diameter of coil and ferrite, and for any ferrite permeability, when the coil is centred on the rod, is:

$$L_f/L_{air} = (1 + x) /(1/k + x/ \mu_{fe}) \qquad 10.1$$

where $x = 5.1 [l' / d_c]/[1+ 2.8 (d_c/ l')]$

$l' = l_c + 0.45 d_c$

$k = [(\varphi/ \varphi_{max} C_{anf} / \varepsilon_0) + 2 d_f] / 2 d_c$

$\varphi/ \varphi_{max} \approx 1 / [1 + \{(l'_f /d_f)^{1.4}\} /(5 \mu_f)]$

$C_{anf} = 0.5 \pi \varepsilon_0 (l_f - l_c) / [Ln \{2 (l_f + d_f)/ d_f \} - 1]$

$\mu_{fe} = (\mu_f -1) (d_f /d_c)^2 +1$

l_c and d_c are the length and diameter of the coil

l_f and d_f are the length and diameter of the ferrite

This equation can be simplified to :

$$L_f/L_{air} \approx x (1+C') \qquad 10.2$$

where $x = 5.1 [l' / d_c]/[1+ 2.8 (d_c/ l')]$

$l' = l_c + 0.45 d_c$

$C' \approx 0.7 [(l_f - l_c) / d_c] / [Ln \{2 (l_f + d_f)/ d_f \} -1]$

This equation should be accurate enough for practical purposes if the antenna meets the following conditions:

a) the coil is no longer than twice its diameter and is centred on the ferrite rod.
a) the inner winding radius of the coil is not too different from that of the ferrite
b) the ferrite permeability is greater than 100
c) the ferrite rod is no longer than 12 times its diameter.

If the ferrite is not centred on the rod then equation A5.1 in Appendix 5 applies.
In applying these equations it should be noted that in the author's experience the permeability of ferrite rods is between a ½ and ¼ of that quoted by the manufacturer. The actual permeability can be determined by measuring the inductance ratio of a coil and ferrite having the same length, and applying Equation 6.1.

Appendix 1: The Inductance of Air Coils

The inductance of a close wound air coil, L_{air}, is given by :

$$L_{air} = [\ \mu_o N^2 A / l_{coil}]\ K_n \qquad \text{A1.1}$$

where N is the number of turns.
A is the area of the coil cross-section
l_{coil} is the length of the coil

K_n is Nagaoka's factor, and is given approximately (Welsby ref 9) as :

$$K_n \approx 1/\ [\ 1+ 0.45\ (d_{coil} / l_{coil}) - 0.005\ (d_{coil} / l_{coil})^2\ ] \qquad \text{A1.2}$$

Where d_{coil} is the coil diameter, and l_{coil} its length

This equation is accurate to within ±1.5% for l_{coil} /d_{coil} from 0.05 to ∞. For l_{coil} /d_{coil} less than 0.05, an exact equation is given by Grover (ref 10), $K_n = 2/\pi\ \ l_{coil} / d_{coil}\ [\text{Ln}\ (4d_{coil} / l_{coil}) - 0.5)$.

If the coil is not close wound, the air inductance calculated by Equation A1.1 must be multiplied by Rosa's factor (below), and it is found that the equations for given here for L_f/L_{air} still apply when the ferrite is inserted into such a loosely wound coil.

$$\text{Rosa's Factor} = 1 - [\ l_{coil}\ (A+B)\ ] / [\pi\ a_{coil}\ N\ K_n\ ] \qquad \text{A1.3}$$

where $A= 2.3\ \text{Log}_{10}\ (1.73\ d_w /p)$,
d_w id the wire diameter
p is the winding pitch (centre to centre)
$B = 0.336\ (1\text{-}2.5/N +3.8/ N^2)$

Appendix 2 : Ratio of Inside to Outside Reluctance

To find the ratio of the inside reluctance to the outside reluctance of an air coil, the equation for the reluctance of an air coil (Payne ref 7) can be used :

$$L= N^2/ (\ \mathcal{R}_{\text{in air}} + \mathcal{R}_{\text{out air}}\ ) \qquad \text{A2.1}$$

where $\mathcal{R}_{\text{in air}} + \mathcal{R}_{\text{out air}} =[1/ \mu_o\ ]\ [l_c / (\pi\ a_c^{\ 2}\ )\ ]\ + [1/ \mu_o\ ]\ [1/(3.49\ a_c)]$
l_c and a_c are the dimensions of the air coil

The ratio of the reluctances is given by the ratio of the two terms above :

$$\mathcal{R}_{\text{in air}} / \mathcal{R}_{\text{out air}} = 3.49/\pi\ \ [l_c / a_c\ ] = 6.98/\pi\ [l_c / d_c\ ] \qquad \text{A 2.2}$$

Experiment indicates that this equation is accurate when l_c / d_c is approximately equal to unity, but deviates at large values of l_c / d_c, and is for instance 80% too low when l_c / d_c =12. This is very surprising given that equation A 2.1 gives <u>very</u> accurate results for the inductance of air-cored coils (ref 7).
Clearly there is an error in A2.1 which becomes apparent only when the *ratio* is taken, and we can anticipate that Equation A 2.1 *should* be of the form :

$$(\mathcal{R}_{\text{in air}} + \mathcal{R}_{\text{out air}}\ ) = [1/ \mu_o\ ]\ [l_c / (\pi\ a_c^{\ 2}\ ) + t\ ]\ + [1/ \mu_o\ ]\ [1/(3.49\ a_c) - t\ ] \qquad \text{A 2.3}$$

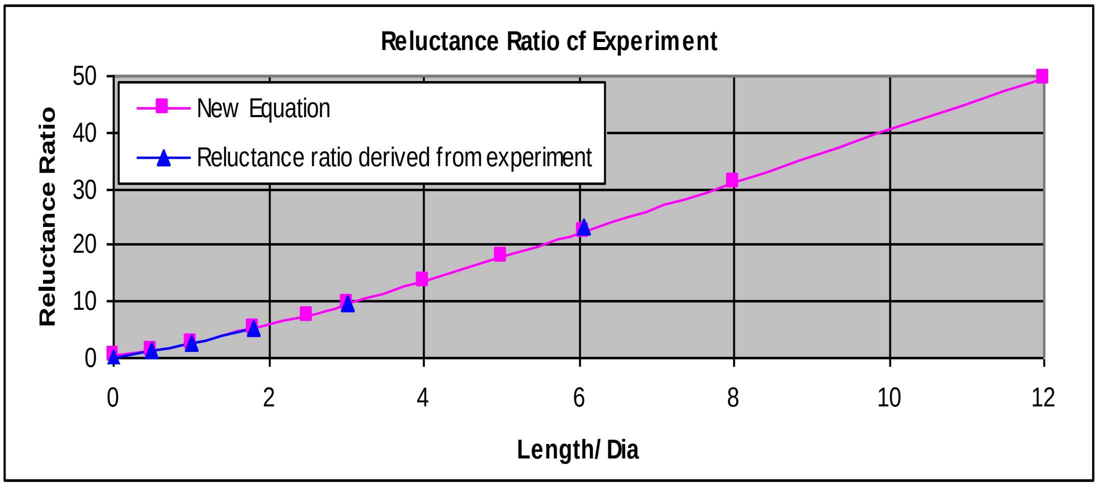
Here a factor t has been added to the first term and subtracted from the second, so that the *sum* is unchanged. However the *ratio* between the two terms will change.

Given the above problem the reluctance ratio was derived from experiments, and these gave the following equation :

$$\mathcal{R}_{\text{in air}} / \mathcal{R}_{\text{out air}} = 5.1\, [l' / d_c]/[1+ 2.8\, (d_c/ l')] \qquad \text{A2.4}$$

$$\text{where } l' = l_c + 0.45\, d_c$$

To determine the above equation the reluctance ratio cannot be measured directly, and must be deduced from measurements of the coil *inductanc*e with and without ferrite, using Equation 6.1. Examination of this equation shows that if a ferrite with a very high permeability is inserted into the coil the inductance will increase by 1+x, where x is the reluctance ratio.
Experiments were conducted with ferrite rods made from Ferroxcube 3C90 material which has a quoted permeability of 2000. Five coils were made (see Appendix 7) having l_c / d_c of 0.5, 1, 1.82, 3.03 and 6.06, and with ferrite of exactly the same length as the coils, gave the following inductance ratios :



For the calculated curve Equation 6.1 was used, with the factors in Equation A2.4 adjusted to obtain a good match with experiment (±4%). The value of ferrite permeability used in this calculation was 1000, half its quoted value. However this permeability *could* have been between say one third of the quoted value up to the quoted value itself, ie from 667 to 2000, but even the lowest value is so high that this had a minimal effect on the calculated values, increasing the error with experiment to only ±3.4 %.
In addition to the experimental points described above there is one theoretical point on the curve which can be calculated, and this is when the ratio l_c / d_c tends to infinity. At that point it can be assumed that the external reluctance tends to $1/(4\, \mu\, d_c)$, by analogy with the capacitance between two one-sided discs which is known to be $4\varepsilon\, d_c$. The internal reluctance will be $\mathcal{R}_{\text{in air}} = l_c /(\mu_1 A) = 4\, l_c /(\mu\, \pi\, d_c^2)$, and so as $l_c / d_c \to \infty$, the value of $\mathcal{R}_{\text{in air}} / \mathcal{R}_{\text{out air}} \to 16/\pi\, l_c / d_c = 5.1\, l_c / d_c$.
It should be noted that it has been assumed that the reluctance ratio conforms to an equation of the form $x= A\, [l_c / d_c]/[(1+ B\, (d_c/ l_c)]$, and the experiments determined the best values for A and B. However there is no theoretical basis for this equation.

The accuracy of Equation A2.4 is difficult to assess. A starting point is to note that the error between experiment and the equation is ±3%, and that the measurement error is probably of the same order, as are the errors in determining the length and diameter of the coils used. These errors are uncorrelated and so are assumed to add in quadrature, giving an overall error of ± 6%. However, Equation A2.4 has been used a number of times in related experiments, and has given predictions which generally agree with experiment to within ±3% (see Figures 6.3, 7.1, and A5.1), and so it is tempting to see this as the accuracy. However, all measurements were with the same equipment and would therefore suffer the same measurement error, so a better estimate is ±4.5%.

Equation A2.4 differs from that derived theoretically in ref 7 for air coils, and it could be significant that this equation has been derived from experiments with ferrite cores. The difference might be due therefore to the field pattern changing when the ferrite is introduced, and then the above equation would apply *only* to ferrite loaded coils, and not air coils. If so then a core of very low permeability would be expected to obey an equation somewhere between that of the Equation 2.4 and that derived theoretically for the air core, Equation A2.2. To test this, cores were made with a permeability of 6.8 and 3.5 using 5 Fair-rite ferrite beads 2643000801 with air gaps (see Appendix 7). A coil was wound with a large l_c / d_c of 6.06, to minimise the errors in the core permeability (see Appendix 7). These experiments agreed with Equations 6.1 and A2.4 to within 15%, which is equal to the experimental error here allowing for the uncertainty on the core permeability. So it appears that Equation A2.4 also applies to the air coil.

Appendix 3 : The Inductance Ratio

The insertion of the ferrite into the coil reduces the reluctance to the flux, and this change in reluctance can be used to determine the change in inductance.

An important consequence of taking the ratio L_f/L_{air} is that now only the *ratio* of the reluctances is required, with and without the ferrite, and not their absolute values. So the ratio L_f/L_{air} is equal to :

$$L_f/L_{air} = \mathcal{R}_{air} / \mathcal{R}_f \qquad \text{A3.1}$$

where $\mathcal{R}_{air}$ is the reluctance of the magnetic path in the air coil
$\mathcal{R}_f$ is the reluctance with the ferrite included

Now the coil has two magnetic paths in series, due to the path inside the coil $\mathcal{R}_{in}$, and that outside $\mathcal{R}_{out}$, so Equation A3.1 becomes :

$$L_f/L_{air} = (\mathcal{R}_{out\ air} + \mathcal{R}_{in\ air}) / (\mathcal{R}_{out\ f} + \mathcal{R}_{in\ f}) \qquad \text{A3.2}$$

Dividing by $\mathcal{R}_{out\ air}$

$$L_f/L_{air} = (1 + \mathcal{R}_{in\ air} / \mathcal{R}_{out\ air}) / [(\mathcal{R}_{out\ f} / \mathcal{R}_{out\ air}) + (\mathcal{R}_{in\ f} / \mathcal{R}_{out\ air})] \qquad \text{A3.3}$$

In the numerator, $\mathcal{R}_{in\ air} / \mathcal{R}_{out\ air}$ is equal to Equation 4.1, and putting this equal to x the numerator becomes (1+x).

Taking the denominator, if the coil is filled internally with ferrite of permeability μ_f , then $(\mathcal{R}_{in\ f} / \mathcal{R}_{out\ air}) = (\mathcal{R}_{in\ air} / \mu_f) / \mathcal{R}_{out\ air}$ and this is equal to x/μ_f.

Also, in the denominator, putting the ratio of the outside reluctance with and without ferrite, $\mathcal{R}_{out\ f} / \mathcal{R}_{out\ air}$ equal to 1/k, then Equation A3.3 becomes :

$$L_f/L_{air} = (1 + x) / (1/k + x/\mu_f) \qquad \text{A3.4}$$

Where x is given by equation 4.1
$1/k = \mathcal{R}_{out\ f} / \mathcal{R}_{out\ air}$
μ_f is the relative permeability of the ferrite

The numerator (1+x) is the sum of the outside reluctance and the inside reluctance, both normalised to the outside reluctance, for the *air* coil. The denominator is the same but with the ferrite present, and the inside reluctance x is seen to be reduced by the relative permeability of the ferrite μ_f. The outside reluctance is reduced by the factor k, and so k is the apparent relative permeability of the outside flux path, and this is addressed in Section 7.

Appendix 4 : Magnetic Current

When the ferrite extends beyond the end of the coil, its effect has been calculated using equations for capacitance in Section 7. However using these for the magnetic circuit assumes that its reluctance is zero (ie infinite permeability), and the inductance will be lower than calculated if the permeability is low and/or the length of the protruding ferrite is long.
The magnetic circuit which needs to be evaluated consists of a ferrite rod with flux continually leaking away from its perimeter as it flows towards its end. This circuit can be approximated by a lumped circuit consisting of loop of the ferrite material of length l /2 with an air gap of length g.
The reluctance of the model is that of the ferrite plus that of the gap :

$$\mathcal{R}_t = l / (2 A \mu_r \mu_o) + g / (A \mu_o) \qquad \text{A4.1}$$

Where A is the cross-sectional area of the ferrite

The reduction in inductance is equal to the ratio of the flux which would flow for a finite permeability μ_r to that which would flow if the permeability was infinite:

$$\Phi / \Phi_{max} = \mathcal{R}_\infty / \mathcal{R}_f = = [g/ (A \mu o)] / [l / (2 A \mu r \mu o) + g/ (A \mu o)]$$

$$\Phi / \Phi_{max} = (2g/ l)\mu r / [1 + (2g/ l)\mu r] \qquad \text{A 4.2}$$

To determine the length of the gap, g, it is assumed that the distributed capacitance of the rod, C_{rod} , is lumped into a parallel plate capacitor of half its total value, (so the model now becomes a ferrite of half the length terminated in half the 'capacitance') :

$$C_{rod}/2 = \varepsilon A/g = \pi \varepsilon a^2/g \qquad \text{A4.3}$$

So

$$2g / l = 4 \pi \varepsilon a^2 / (l C_{rod}) \qquad \text{A 4.4}$$

Combining this equation with Equation A4.2 gives the flux ratio as :

$$\Phi / \Phi_{max} = (2g/ l)\mu r / [1 + (2g/ l)\mu r] \qquad \text{A 4.5}$$

where $2g / l = 4 \pi \varepsilon a^2 / (l C_{rod})$

This can be re-written as :

$$\Phi / \Phi_{max} = 1/(1+\Psi)$$

where $\Psi = (l C_{rod}) / (K_a 4 \pi \varepsilon a^2 \mu_r)$

The length l in the above equation is the length of ferrite protruding from the coil. K_a is a factor chosen to provide better agreement with experiment, recognizing that the above analysis is approximate. Best agreement with experiment is with K_a =4.
The above equation can be simplified, since $C_{rod} \approx 19 \varepsilon [l /(2a)]^{0.6}$ to the accuracy required here, and overall good agreement with experiment is provided by :

$$\boldsymbol{\Phi / \Phi_{max} \approx 1 / [1 + (l'_f / d_f)^{1.4} / (5 \mu_r)]} \qquad \textbf{A 4.6}$$

where l'_f is the protruding length ($l_f - l_{coil}$)

This equation has been tested against measurements with two combinations of ferrite length, radius, and permeability, as follows :

	Exp1	Exp 2	Ratio
Core Diameter	9.42	6.87	1.4

Core Area			1.9
Core Length/D	25	11	2.3
Core Permeability	67	3.6	19
Crod	135	56.7	2.4
Φ / Φ_{max}	0.72	0.39	

The above equation gave values of inductance ratio which agreed with measurements to within ±3% in both cases, equal to the experimental error.
Notice that in experiment 2 the effective permeability was only 3.6, and yet the above equation accurately predicted the reduction of flux to 0.39. This low permeability was provided by iron powder as described in Appendix 7.

Appendix 5 : Coil Offset from Centre

In broadcast receivers there are often two coils on the rod, one coil for MW reception and another for LW reception. Each coil is therefore offset from the centre of the rod.
The following analysis assumes that there is no interaction between the two coils, so for instance when one is in use the other is open circuited.
Taking one of these coils its *centre* will be a distance l_{f1} from one end of the rod and a distance l_{f2} from the other end, so that the overall length of the rod is ($l_{f1} + l_{f2}$). The overall capacitance between these two unequal length cylinders is equal to the series capacitance of two unequal monopole antennas, and given by:

$$C_{anf\ combined} = (2C_1)(2C_2)/(2C_1 + 2C_2) = 2C_1C_2/(C_1 + C_2) \qquad \textbf{A5.1}$$

The capacitances C_1 and C_2, are the dipole capacitances calculated from the sum of Equations 7.3 and 7.5 for ferrite lengths of $2l_{f1}$ and $2l_{f2}$ respectively.
Equation A5.1 is now used in place of [C_{anf} + $C_{end\ f}$] for the calculation of k.

Calculations of the inductance with offset, along with measurements are given below. The coil had a length of 20mm, wound with 75 turns of fine wire with a winding diameter to the centre of the wire of 11.2 mm. The ferrite had length of 120 mm and a diameter of 9.42 mm.

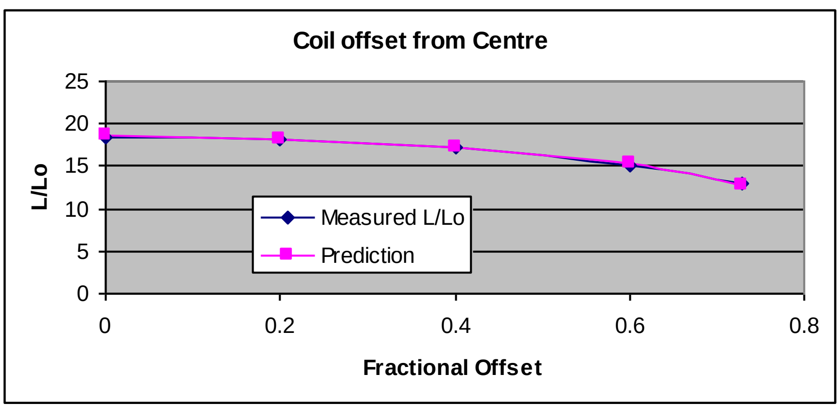


Figure A5.1 Inductance of Coil Offset from Centre of Ferrite Rod

The theoretical curve above is within ±3% of the measured values over the range displayed, for the measured the rod permeability of 90.

Appendix 6 : Measurements

The inductance was measured at 1Mhz using an Array Solutions AIM 4170 Analyser, with the jig calibrated-out and the analyser set to average 16 measurements at each frequency, to minimise quantisation errors and noise. For each measurement the self resonant frequency f_r of the coil, or coil + ferrite was also measured, and this was used to correct the measured inductance L_c using the following equation (see Welsby ref 5, p 37)

$$L = L_c [1- (f / f_r)^2] \qquad \text{A 6.1}$$

This correction is accepted practice and is based on the assumption that coils have a self capacitance, and that this increases the apparent inductance. However the author has recently published an article which shows that the coil is a transmission-line and the change in inductance with frequency is a real change, and that the above factor should therefore be part of the *theory*, rather than a correction of the measurements (Payne ref 8). However for the purpose of validating the theory it doesn't matter particularly whether this 'correction' is applied to the theory or the measurements, and since the latter is accepted practice this was done here.

Appendix 7 : Coils and Ferrites

Coils

Coils were made to evaluate the accuracy of the equations here. These needed to be made of larger diameter than the ferrite so that separate measurements could be made of the coil without ferrite and with (see Measurements above). In principle any coil diameter larger than that of the ferrite could be used, as long as a correction was applied for the effective permeability using Equation 5.2.1. However this equation is not accurate when the diameters differ greatly because flux then flows in the air gap between the ferrite and the winding and this is not accounted-for in this equation. So it was important that the coils were a close fit over the ferrite, but not so close that they would not slide easily over the ferrite rod, thus allowing the inductance to be measured both with and without ferrite. A very successful method of winding these coils was to wind the wire tightly over the ferrite, release it slightly so that it partially unwound, and the windings then held in place by Sellotape over the outside of the winding. Such a winding is shown below :

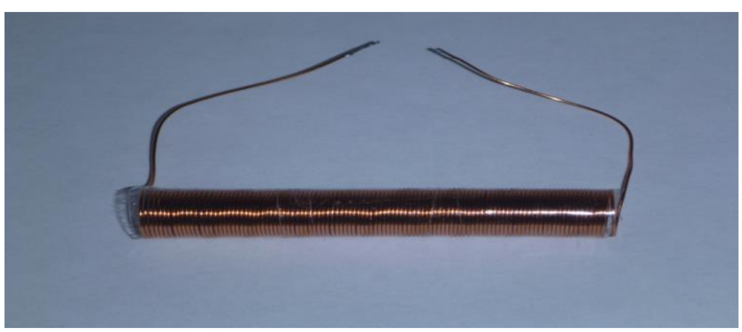


Figure A7.1 A coil wound as per text

Ferrite

Most of the ferrite rods used were as supplied by the manufacturers, and generally had a diameter between 7.8 and 10 mm, and lengths from 25 to 120 mm. Sometimes other lengths were required and then a rod was cut with a Dremel high speed tool with a diamond tipped saw. This cutting could have degraded the ferrite since it known that some very high frequency ferrites are sensitive to mechanical stress. However no evidence was found for this in the low frequency ferrites used here.

All rods were NiZn with permeability between 325 and 2000, as quoted by the manufacturer. However there is strong evidence presented here that the permeability of these rods was between ¼ and ½ these values.

Low Permeability using Ferrite Beads

The reduction in magnetic current (Equation 8.1) is not very large for normal values of ferrite permeability and rod length, and so to test this equation a rod of low permeability was need, having a value in the range 2 to 10. Materials with this range of permeability are not readily available, and it was therefore necessary to fabricate them. Two methods were tried :

The first method used a chain of ferrite beads with air-gaps between them, the gaps being formed by fibre washers. Such a construction is shown below :

Figure A 7.2 Low Permeability Composite Rod

The effective permeability μ_{eff} will then be equal to the ratio of the reluctance of the core without the ferrite to that with the ferrite (and gaps). Since $\mathcal{R} = l/(\mu_o \mu_r A)$, and A is constant, this ratio becomes :

$$\mu_{eff} = l_{total} / [l_{gap} + (l_{ferrite}/\mu_r)]$$

$$= [l_{gap} + l_{ferrite}] / [l_{gap} + (l_{ferrite}/\mu_r)] \qquad 1.5.1$$

where l_{gap} is the length of the air gap
$l_{ferrite}$ is the length of the ferrite

The beads were made of Fair-rite material 43, which has a quoted permeability of 800 (up 1 MHz), and since the sum of the gaps was never longer than the sum of the beads, the denominator $l_{gap} + (l_{ferrite}/\mu_r)]$ is approximately equal to l_{gap} to within a fraction of 1%. This still holds if the bead permeability was somewhat less than that quoted. So the above equations can be simplified to :

$$\mu_{eff} \approx 1 + [l_{ferrite} / l_{gap}] \qquad 1.5.2$$

Or, transposing :

$$[l_{ferrite} / l_{gap}] \approx \mu_{eff} - 1 \qquad 1.5.3$$

So for an effective permeability of 3 each air-gap in the chain of ferrite beads must be half the length of each bead.

To minimise any effects from the 'lumped' nature of this rod, the beads chosen had a length no longer their diameter (even shorter beads would have been an advantage but none were found).

The advantage of this technique is that the average permeability can be calculated accurately, because the length of the ferrite and that of the gaps can be measured accurately. Also if the ferrite permeability is very high, any uncertainty here is negligible since the overall permeability is essentially determined by the relative length of the gaps. However there is uncertainty at each end – (is the end air-gap or ferrite ?). So the accuracy of the effective permeability is no better than ±1/(2N) where N is the number of beads, and in practice therefore at least 5 beads are required.

Low Permeability Core using Iron Powder

The second method to produce low permeability cores, was to use iron powder. This is readily available in pure form with the particle size classified as Mesh 100 ie mesh size 0.152mm. The particles were grey in colour and so showed no obvious signs of having a corrosion layer. To form a long 'rod' these were poured into a plastic tube, and a coil wound around the outside. The tube was mounted vertically so that the only compacting force on the powder was gravity.

To determine the permeability of this powder it was poured into the hollow former of a coil as above, but only over a length equal to that of the coil. The inductance was measured both before filling and after filling and gave a ratio of 3.09. From Equation 6.1 this would be achieved with an effective permeability of 5.2, and allowing for the ratios of the radii of coil and iron filling $(10.9/10.5)^2$, this gave a permeability of the iron powder of 5.6 (for gavity force).

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Issue 2 : June 2014 : Equation 4.1 modified to give better accuracy for short coils. Appendix 2 incorporates the measurements and theoretical calculation for this change. Also errors corrected in Equations A1.2 and A1.3

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